Homework 5 Solutions

§4.1

1/ $\frac{dy}{dx} = \frac{d}{dx}(12x^3 - 8x^2 + 7x + 5)$

$= 12\frac{d}{dx}(x^3) - 8\frac{d}{dx}(x^2) + 7\frac{d}{dx}(x) + \frac{d}{dx}(5)$

$= 12.3 \cdot x^2 - 8.2 \cdot x + 7 \cdot 1$

$= 36x^2 - 16x + 7$

7/ $\frac{dy}{dx} = \frac{d}{dx}(8\sqrt{x} + 6x^{3/4})$

$= 8 \cdot \frac{1}{2}x^{-\frac{1}{2}} + 6 \cdot \frac{3}{4}x^{-1/4}$

12/ $f(t) = \frac{14}{t} + \frac{12}{t^4} + \sqrt{2} \Rightarrow f'(t) = -14t^{-2} + 12 \cdot (-4)t^{-5}$

20/ $g(x) = \frac{x^3 - 4x}{\sqrt{x}} = x^{5/2} - 4x^{1/2}$

$\Rightarrow g'(x) = \frac{5}{2}x^{3/2} - 4 \cdot \frac{1}{2}x^{-\frac{1}{2}}$

24/ (a)

30/ $f(x) = \frac{x^3}{9} - 7x^2 \Rightarrow f'(x) = \frac{3x^2}{9} - 7 \cdot 2 \cdot x$

$\Rightarrow f'(3) = \frac{3 \cdot 3^2}{9} - 7 \cdot 2 \cdot 3 = -39$

37/ $f(x) = 2x^3 + 9x^2 - 60x + 4$

$\Rightarrow f'(x) = 6x^2 + 18x - 60 = 6(x^2 + 3 - 10) = 6(x+5)(x-2)$

Horizontal Tangent $\Rightarrow f'(x) = 0 \Rightarrow 6(x+5)(x-2) = 0 \Rightarrow x = 2, -5.$

45/ $f(x) = \frac{1}{2}g(x) + \frac{1}{4}h(x)$

$\Rightarrow f'(x) = \frac{1}{2}g'(x) + \frac{1}{4}h'(x) \Rightarrow f'(2) = \frac{1}{2}g'(2) + \frac{1}{4}h'(2) = \frac{1}{2} \cdot 7 + \frac{1}{4} \cdot 14$
$= 7$

51/ $q = 5000 - 100p$

$\Rightarrow p = 50 - \frac{1}{100}q$

Marginal Revenue

$\Rightarrow R = pq = (50 - \frac{1}{100}q)q = 50q - \frac{1}{100}q^2 \Rightarrow \frac{dR}{dq} = 50 - \frac{2}{100}q$

a) Marginal Revenue when $q = 1000 = 50 - \frac{2 \cdot 1000}{100} = 30$

b) Marginal Revenue when $q = 2500 = 50 - \frac{2 \cdot 2500}{100} = 0$

c) Marginal Revenue when $q = 3000 = -10.$

53/ $p = \frac{1000}{q^2} + 1000$

$\Rightarrow R = pq = \left(\frac{1000}{q^2} + 1000\right) q = \frac{1000}{q} + 1000q$

$\Rightarrow R'(q) = \frac{-1000}{q^2} + 1000$

$R'(10) = \frac{-1000}{10^2} + 1000 = 990.$

60/ a) $G(0) = 450$, b) $G(25) = 325$

$G'(x) = (-0.2).2.x$

c) $G'(10) = -4$, d) $G'(25) = -10$

62/ a) $m(30) = \frac{(30)^3}{100} - \frac{1500}{30} = 220$ grams

b) $m'(c) = \frac{3c^2}{100} + \frac{1500}{c^2}$

$m'(30) = 28\frac{2}{3}$ grams per cm.

When the circumference is 30 cm its mass is increasing at a rate of $28\,^2/_3$ grams per centimeter.

§4.2

7/ $y = uv$, $u = x+1$, $v = \sqrt{x} + 2$

$\Rightarrow \frac{du}{dx} = 1$, $\frac{dv}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}v + u.\frac{dv}{dx} = 1.(\sqrt{x}+2) + (x+1)(\frac{1}{2}x^{-\frac{1}{2}})$

10/ $q = uv$, $u = x^{-2} - x^{-3}$, $v = 3x^{-1} + 4x^{-4}$

$\Rightarrow \frac{du}{dx} = -2x^{-3} + 3x^{-4}$, $\frac{dv}{dx} = -3x^{-2} - 16x^{-5}$

$\Rightarrow \frac{dq}{dx} = \frac{du}{dx}v + u.\frac{dv}{dx} = (-2x^{-3} + 3x^{-4})(3x^{-1} + 4x^{-4}) + (x^{-2} - x^{-3})(-3x^{-2} - 16x^{-5})$

13/ $y = \frac{u}{v}$, $u = 5 - 3t$, $v = 4 + t$

$\Rightarrow \frac{du}{dt} = -3$, $\frac{dv}{dt} = 1$

$$\Rightarrow \frac{dy}{dt} = \frac{\frac{du}{dt} v - u \cdot \frac{dv}{dt}}{v^2}$$

$$= \frac{-3(4+3) - (5-3t) \cdot 1}{(4+t)^2}$$

21/ $p = \frac{u}{v}$, $u = \sqrt{t}$, $v = t - 1$

$\Rightarrow \frac{du}{dt} = \frac{1}{2} t^{-\frac{1}{2}}$, $\frac{dv}{dt} = 1$

$$\Rightarrow \frac{dp}{dt} = \frac{\frac{du}{dt} v - u \cdot \frac{dv}{dt}}{v^2} = \frac{(\frac{1}{2} t^{-\frac{1}{2}})(t-1) - (\sqrt{t}) \cdot 1}{(t-1)^2}$$

27/ $f = \frac{u}{v}$, $u = (3x^2+1)(2x-1) = 6x^3 - 3x^2 + 2x - 1$
$v = 5x + 4$

$\Rightarrow \frac{du}{dx} = 18x^2 - 6x + 2$, $\frac{dv}{dx} = 5$

$$\Rightarrow \frac{df}{dx} = \frac{\frac{du}{dx} v - u \cdot \frac{dv}{dx}}{v^2}$$

$$= \frac{(18x^2 - 6x + 2)(5x+4) - (6x^3 - 3x^2 + 2x - 1) \cdot 5}{(5x+4)^2}$$

33/ $f = \frac{u}{v}$, $u = x$, $v = x - 2$

$\Rightarrow \frac{du}{dx} = \frac{dv}{dx} = 1$

$$\Rightarrow f'(x) = \frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2} = \frac{1 \cdot (x-2) - x \cdot 1}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

$f(3) = 3$
$f'(3) = -2$
$\Rightarrow$ Tangent has equation
$(y-3) = -2(x-3)$

41/ a) $f(x) = x^2 + bx \quad \Rightarrow \quad f'(x) = 2x + b \quad \Rightarrow \quad f'(x)g'(x) = (2x+b)c$

$g(x) = cx + d \qquad\qquad g'(x) = c \qquad\qquad\qquad = 2cx + bc$

b) $f(x)g(x) = cx^3 + (d+bc)x^2 + bdx$

$$\Rightarrow \frac{d\,f(x)g(x)}{dx} = 3cx^2 + 2(d+bc)x + bd$$

For general b, c, d these are not equal.

E.g. $c = d = b = 1$

45/ a) $M = \frac{u}{v}$, $u = 100d^2$, $v = 3d^2 + 10$

$$\Rightarrow \frac{du}{dd} = 200d, \quad \frac{dv}{dd} = 6d$$

$$\Rightarrow \frac{dM}{dd} = \frac{\frac{du}{dd}v - u \cdot \frac{dv}{dd}}{v^2}$$

$$= \frac{200d(3d^2+10) - 100d^2 \cdot 6d}{(3d^2+10)^2}$$

b) $M'(2) = 8.3$: The new employee can assemble about 8.3 additional bikes per day after 2 days of training

$M'(5) = 1.4$: The new employee can assembe about 1.4 additional bikes per day after 5 days of training

50/ a) $S = \frac{u}{v}$, $u = x$, $v = m + nx$

$$\Rightarrow \frac{du}{dx} = 1, \quad \frac{dv}{dx} = n$$

$$\Rightarrow \frac{dS}{dx} = \frac{\frac{du}{dx}v - u \cdot \frac{dv}{dx}}{v^2} = \frac{1 \cdot (m+nx) - x \cdot n}{(m+nx)^2} = \frac{m}{(m+nx)^2}$$

b) $$S'(50) = \frac{10}{(10 + 3.50)^2}$$

54/ $f = \frac{u}{v}$, $u = 90t$, $v = 99t - 90$

$\Rightarrow \frac{du}{dt} = 90$, $\frac{dv}{dt} = 99$

$$\Rightarrow \frac{df}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2} = \frac{90(99t-90) - 90t\cdot 99}{(99t-90)^2} = \frac{-8100}{(99t-90)^2}$$

a) $f'(1) = -100$, b) $f'(10) = -0.01$

§4.3/

6/ $g(f(5z)) = 8(f(5z)) + 3$

$= 8\left(5(5z)^2 - 2(5z)\right) + 3$

$= 1000z^2 - 80z + 3$

7/ $g(f(x)) = 6f(x) - 1 = 6\left(\frac{x}{8} + 7\right) - 1$

$f(g(x)) = \frac{g(x)}{8} + 7 = \frac{6x-1}{8} + 7$

19/ $y = f(g(x))$

$g(x) = x^2 + 5x$

$f(x) = x^{1/3} - 2x^{2/3} + 7$

28/ $f = 8\sqrt{u}$, $u = 4t^2 + 7$

$\Rightarrow \frac{df}{du} = 8\cdot\frac{1}{2}u^{-\frac{1}{2}} = 8\cdot\frac{1}{2}(4t^2+7)^{-\frac{1}{2}}$

$\frac{du}{dt} = 8t$

$\Rightarrow \frac{df}{dt} = \frac{df}{du}\cdot\frac{du}{dt} = 8\cdot\frac{1}{2}(4t^2+7)^{-\frac{1}{2}}\cdot 8t$

31/ $y = uv$, $u = (3x^4+1)^4$, $v = x^3 + 4$

$\Rightarrow u = z^4$, $z = 3x^4 + 1$

$\Rightarrow \frac{du}{dx} = \frac{du}{dz}\cdot\frac{dz}{dx} = 4z^3\cdot 12x^3 = 4(3x^4+1)^3\cdot 12x^3$

$\frac{dv}{dx} = 3x^2$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$
$$= \left(4(3x^4+1)^3 \cdot 12x^3)(x^3+4)\right) + \left((3x^4+1)^4 \cdot 3x^2\right)$$

48/ $f = uv$, $u = x^2$, $v = \sqrt{x^4-12}$

$\Rightarrow v = \sqrt{z}$, $z = x^4 - 12$

$$\Rightarrow \frac{dv}{dx} = \frac{dv}{dz} \cdot \frac{dz}{dx} = \frac{1}{2}z^{-\frac{1}{2}} \cdot 4x^3 = \frac{1}{2}(x^4-12)^{-\frac{1}{2}} \cdot 4x^3$$

$$\frac{du}{dx} = 2x$$

$$\Rightarrow f'(x) = \frac{du}{dx}v + u \cdot \frac{dv}{dx}$$
$$= 2x\sqrt{x^4-12} + x^2 \cdot \frac{1}{2}(x^4-12)^{-\frac{1}{2}} \cdot 4x^3$$

$$\Rightarrow f'(2) = 2 \cdot 2\sqrt{16-12} + 2^2 \cdot \frac{1}{2}(16-12)^{-\frac{1}{2}} \cdot 4 \cdot 2^3$$
$$= 40$$

$$f(2) = 2^2 \cdot \sqrt{16-12} = 8$$

$\Rightarrow$ Tangent line has equation $y - 8 = 40(x-2)$

50/ $f = \frac{u}{v}$, $u = x$, $v = (x^2+4)^4$

$\Rightarrow v = z^4$, $z = x^2+4$

$$\Rightarrow \frac{dv}{dx} = \frac{dv}{dz} \cdot \frac{dz}{dx} = 4z^3 \cdot 2x = 4(x^2+4)^3 \cdot 2x$$

$$\frac{du}{dx} = 1$$

$$\Rightarrow f'(x) = \frac{\frac{du}{dx}v - u \cdot \frac{dv}{dx}}{v^2} = \frac{1 \cdot (x^2+4)^4 - x \cdot 4(x^2+4)^3 \cdot 2x}{(x^2+4)^8}$$
$$= \frac{x^2+4 - 8x^2}{(x^2+4)^5}$$
$$= \frac{-7x^2+4}{(x^2+4)^5}$$

$$f'(x) = 0 \Rightarrow -7x^2+4 = 0 \Rightarrow x = \pm\sqrt{\frac{4}{7}}$$

55/ $A = 1500\, z^{1825}, \quad z = 1 + \frac{r}{36500}$

$$\Rightarrow \frac{dA}{dr} = \frac{dA}{dz} \cdot \frac{dz}{dr} = 1500 \cdot 1825 \cdot z^{1824} \cdot \frac{1}{36500}$$

$$= 1500 \cdot 1825 \cdot \left(1 + \frac{r}{36500}\right)^{1824}$$

a) $A'(6) = 101.22$

b) $A'(8) = 111.86$

c) $A'(9) = 117.59$

§4.4

7/ $\frac{d}{dx}(x^2) = 2x \Rightarrow \frac{d}{dx}\left(e^{(x^2)}\right) = e^{(x^2)} \cdot 2x$

19/ $y = \frac{u}{v}, \quad u = e^{x} + e^{(-x)}, \quad v = x$

$$\Rightarrow \frac{du}{dx} = e^{x} + e^{(-x)} \cdot (-1) = e^{x} - e^{-x}$$

$$\frac{dv}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2} = \frac{(e^{x} - e^{-x})\, x - (e^{x} + e^{-x}) \cdot 1}{x^2}$$

21/ $P = \frac{10000}{(9 + 4e^{-0.2t})} = 10000\, u^{-1}, \quad u = 9 + 4e^{(-0.2t)}$

$$\Rightarrow \frac{dP}{dt} = \frac{dP}{du} \cdot \frac{du}{dt} = -10000\, u^{-2} \cdot 4e^{-0.2t} \cdot (-0.2)$$

$$= -10000\, (9 + 4e^{-0.2t})^{-2} \cdot 4e^{-0.2t} (-0.2)$$

37/ $y = \frac{u}{v}, \quad u = te^{t} + 2, \quad v = e^{2t} + 1$

$$\frac{du}{dt} = \frac{d}{dt}(te^{t}) + \frac{d}{dt}(2) = \frac{d}{dt}(te^{t})$$

$$= \frac{d}{dt}(t)\, e^{t} + t \frac{d}{dt}(e^{t}) = e^{t} + te^{t}$$

$$\frac{dv}{dt} = \frac{d}{dt}(e^{2t} + 1) = \frac{d}{dt}(e^{2t}) + \frac{d}{dt}(1) = \frac{d}{dt}(e^{2t}) = e^{2t} \cdot 2$$

$$\Rightarrow \frac{dy}{dt} = \frac{\frac{du}{dt} v - u \cdot \frac{dv}{dt}}{v^2} = \frac{(e^t + te^t)(e^{2t}+1) - (te^t+2) \cdot e^{2t} \cdot 2}{(e^{2t}+1)^2}$$

39/ $C = \sqrt{u}$, $u = 400 - 800\, 1.1^{(-x)}$

$$\Rightarrow \frac{dc}{du} = \frac{1}{2} u^{-\frac{1}{2}} = \frac{1}{2}(400 - 800\, 1.1^{(-x)})^{-\frac{1}{2}}$$

$$\frac{du}{dx} = -800 \frac{d}{dx}(1.1^{(-x)}) = -800 \ln(1.1)\, 1.1^{(-x)} \cdot (-1)$$

$$\Rightarrow \frac{dC}{dx} = \frac{dc}{du} \cdot \frac{du}{dx} = \frac{1}{2}(400 - 800\, 1.1^{(-x)})^{-\frac{1}{2}} (-800) \ln(1.1)\, 1.1^{(-x)} (-1)$$

a) $c'(0) = 3.81$

b) $C'(20) = 0.2$

c) $\lim_{x \to \infty} C'(x) = 0$

44/

$$S = 5000\, e^{(0.1 e^{(0.25t)})}$$

$$\Rightarrow \frac{dS}{dt} = 5000\, e^{(0.1 e^{(0.25t)})} \cdot \frac{d}{dt}(0.1\, e^{(0.25t)})$$

$$= 5000\, e^{(0.1 e^{(0.25t)})} \cdot (0.1)\, e^{(0.25t)} \cdot 0.25$$

$$S'(8) = 5000\, e^{(0.1 e^2)} \cdot (0.1) \cdot e^2 \cdot (0.25)$$

$$\approx 1934 \qquad \underline{\underline{(b)}}$$

54/ $\frac{dR}{dt} = ?$ when $c = 180$, $\frac{dc}{dt} = 15$

$$\frac{dR}{dt} = \frac{dR}{dc} \cdot \frac{dc}{dt} = 3.14 \cdot \ln(1.006)\, 1.006^c \cdot \frac{dc}{dt}$$

$$= 3.14 \ln(1.006)\, 1.006^{180} \cdot 15 \approx 0.84$$

56/ $\frac{dH}{dN} = \frac{d}{dN}\left(1000 - 1000e^{-kN}\right)$

$= -1000 \frac{d}{dN} e^{(-kN)} = -1000\, e^{(-kN)} \cdot (-k)$

$= 1000\, k e^{-kN}$

$k = 0.1 \Rightarrow H'(N) = 10 e^{-0.1N}$

a) $H'(10) \simeq 36.8$

b) $H'(100) \simeq 0.00454$

c) $H'(1000) = 10\, e^{-100}$ very small

d) $e^{-0.1N} > 0 \Rightarrow H'(N) > 0$

$\Rightarrow$ This means the strength of a habit always increases the more it is repeated.