Homework 4 Solutions

§3.4 The Definition of the Derivative

1/ a) 0, b) 1, c) −1, d) DNE

2/ a) DNE, b) at $x=0$ tangent is vertical and vertical lines have undefined slope

3/ Rational Functions are differentiable at all points in domain. Hence $\frac{x^2-1}{x+2}$ is not differentiable only at $x=-2$.

4/ The tangent line is horizontal.

16/ $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$$

$$= \lim_{h\to 0} \frac{3x - 3(x+h)}{hx(x+h)} = \lim_{h\to 0} \frac{-3}{x(x+h)} = \frac{-3}{x^2}$$

$f'(-2) = \frac{-3}{(-2)^2}$, $f'(0)$ DNE, $f'(3) = \frac{-3}{3^2}$

18/ $$f'(x) = \lim_{h\to 0} \frac{-3\sqrt{x+h} - (-3\sqrt{x})}{h}$$

$$= \lim_{h\to 0} \frac{-3(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{-3}{(\sqrt{x+h}+\sqrt{x})} = \frac{-3}{2\sqrt{x}}$$

$f'(-2)$ DNE, $f'(0)$ DNE, $f'(3) = \frac{-3}{2\sqrt{3}}$

19/ $$f'(x) = \lim_{h\to 0} \frac{(2(x+h)^3+5) - (2x^3+5)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^3} + 6xh^2 + 6x^2h + 2h^3 \cancel{+5} - \cancel{2x^3} \cancel{-5}}{h}$$

$$= \lim_{h\to 0} 6xh + 6x^2 + 2h^2 = 6x^2$$

$f'(-2) = 6(-2)^2$, $f'(0) = 0$, $f'(3) = 6\cdot 3^2$

22/ Secant : $y - 5 = \frac{-3-5}{3-(-1)}(x-(-1))$

$$f'(x) = \lim_{h\to 0} \frac{(6-(x+h)^2) - (6-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{6} - \cancel{x^2} - 2xh - h^2 - \cancel{6} + \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} -2x - h = -2x$$

$f'(-1) = -2 \cdot -1 = 2$

Tangent : $y - 5 = 2(x+1)$

24/ Secant : $y - (\frac{-3}{2}) = \frac{\frac{-3}{6} - \frac{-3}{2}}{5-1}(x-1)$

$$f'(x) = \lim_{h\to 0} \frac{\frac{-3}{(x+h+1)} - \frac{-3}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{-3(x+1) - (-3)(x+h+1)}{h(x+1)(x+h+1)}$$

$$= \lim_{h\to 0} \frac{3h}{h(x+1)(x+h+1)} = \lim_{h\to 0} \frac{3}{(x+1)(x+h+1)} = \frac{3}{(x+1)^2}$$

$f'(1) = \frac{3}{2^2}$

Tangent : $y - (\frac{-3}{2}) = \frac{3}{2^2}(x-1)$

35/ $x = 0$

36/ $x = -6$

37/ $x = -3, 1, 0, 2, 3, 5$

39/ a) $(a, 0)$ and (b, c)
b) $(0, b)$
c) $x = 0$, $x = b$

49/ a) $D'(p) = \lim_{h \to 0} \dfrac{(-2(p+h)^2 - 4(p+h) + 300) - (-2p^2 - 4p + 300)}{h}$

$= \lim_{h \to 0} \dfrac{-4ph - 2h^2 - 4h}{h} = \lim_{h \to 0} -4p - 4 - 2h = -4p - 4$

b) $D'(10) = -44$. Demand is decreasing at a rate of about 44 items for each increase of \$1.

56/ $I'(t) = \lim_{h \to 0} \dfrac{27 + 72(t+h) - 1.5(t+h)^2 - 27 - 72t + 1.5t^2}{h}$

$= \lim_{h \to 0} \dfrac{72h - 3th - 1.5h^2}{h} = \lim_{h \to 0} 72 - 3t - 1.5h = 72 - 3t$

a) $I'(5) = 72 - 15 = 57$

The rate of change of food intake 5 mins into meal is 57 grams per minute

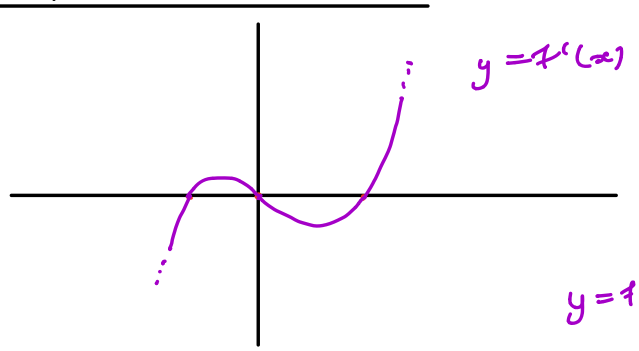
b) $I'(t) = 0 \Rightarrow 72 - 3t = 0 \Rightarrow t = 24$ mins

c) After 24 mins the meal is finished.

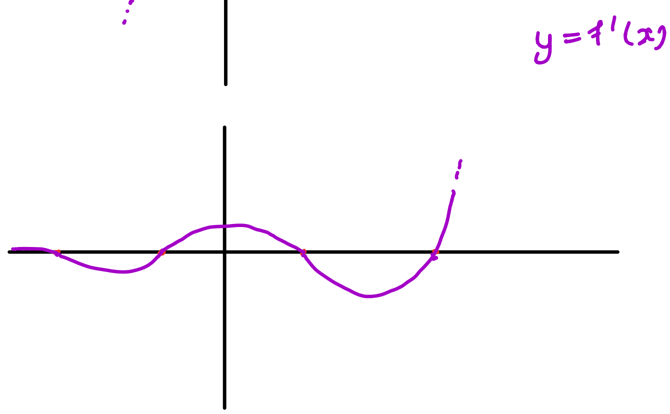
Logical domain is $[0, 24]$.

§3.5 Graphical Differentiation

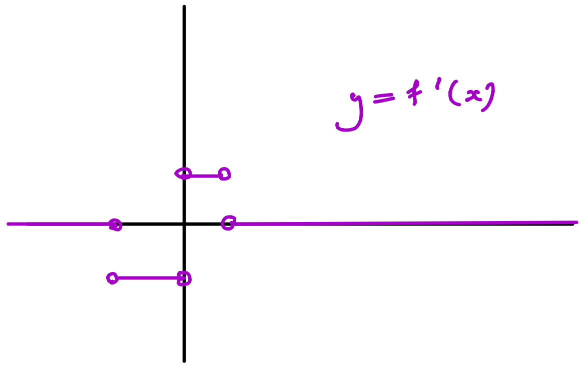
7/



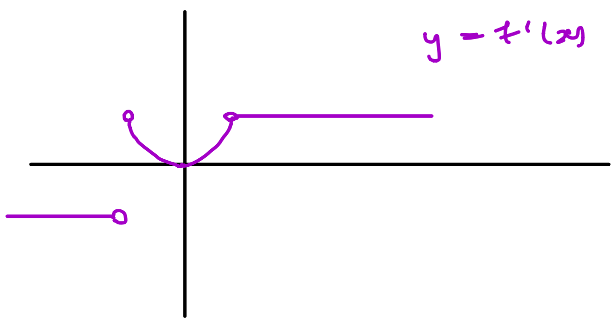
8/



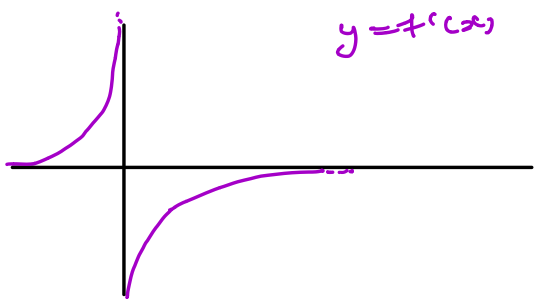
9/



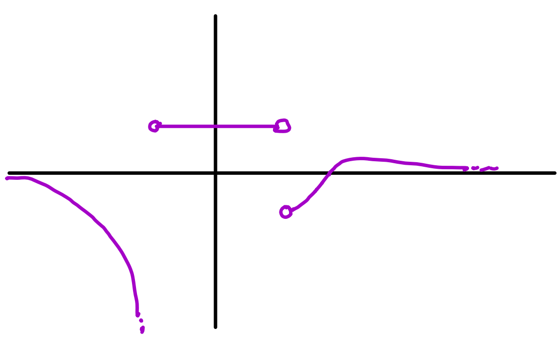
11/



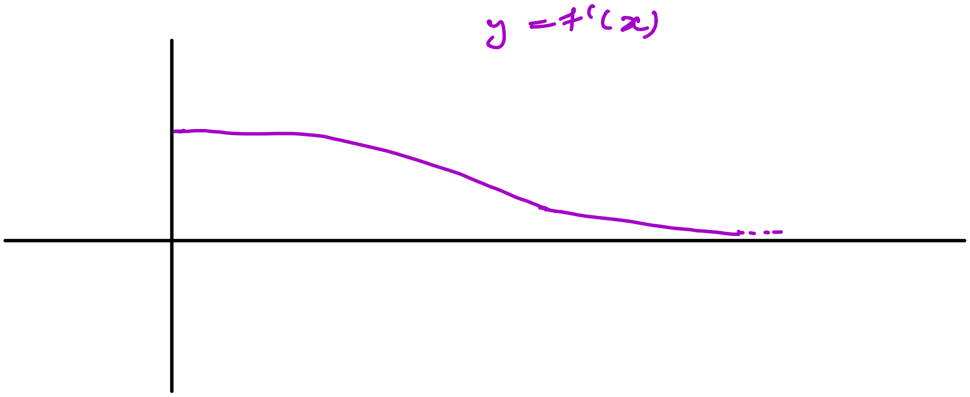
13/



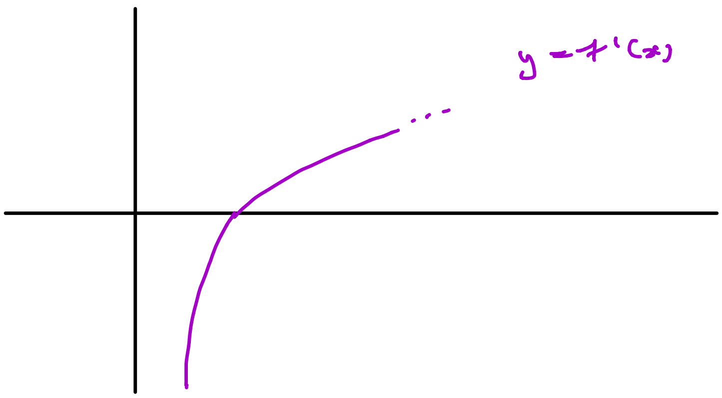
16/



19/



20/



24/