Homework 3 Solutions

§3.1 Limits

43/ $\deg(3x) = \deg(7x-1) = 1 \Rightarrow \lim_{x\to\infty} \frac{3x}{7x-1} = \frac{3}{7}$

45/ $\deg(3x^2+2x) = \deg(2x^2-2x+1) = 2 \Rightarrow \lim_{x\to-\infty} \frac{3x^2+2x}{2x^2-2x+1} = \frac{3}{2}$

48/ $\deg(2x^2-1) = 2 < \deg(3x^4+2) = 4 \Rightarrow \lim_{x\to\infty} \frac{2x^2-1}{3x^4+2} = 0$

50/ $\lim_{x\to\infty} \frac{x^4 - x^3 - 3x}{7x^2+9} = \lim_{x\to\infty} \frac{x^4}{7x^2} = \lim_{x\to\infty} \frac{1}{7}x^2 = \infty$ (DNE)

52/ $\lim_{x\to\infty} \frac{-5x^3-4x^2+8}{6x^2+3x+2} = \lim_{x\to\infty} \frac{-5x^3}{6x^2} = \lim_{x\to\infty} \frac{-5}{6}x = -\infty$ (DNE)

64/ a) $\lim_{x\to 4} -6 = -6 < 0$

$\lim_{x\to 4} (x-4)^2 = (4-4)^2 = 0$

$\frac{4}{(x-4)^2>0 \quad (x-4)^2>0} \Rightarrow \lim_{x\to 4} (x-4)^2 = 0^+$

$\Rightarrow \lim_{x\to 4} \frac{-6}{(x-4)^2} = -\infty$ (DNE)

b) $x=4$ is the only vertical asymptote

c) Yes. $x=a$ is a vertical asymptote $\Leftrightarrow \lim_{x\to a^+/a^-/a} f(x) = \pm\infty$

69/ a) $\lim_{x\to 0^+} \ln(x) = -\infty$

b) Vertical asymptote at $x=0$

87/ $C(x) = 0.0417x + 167.55$

$\Rightarrow \overline{C}(x) = \frac{0.0417x + 167.55}{x}$ ← rational with degree 1 top and bottom

$\Rightarrow \lim_{x\to\infty} \overline{C}(x) = 0.0417$ = cost per mile as the number of miles becomes very large.

91/ $0<g<i \Rightarrow 0 < \frac{1+g}{1+i} < 1 \Rightarrow \lim_{n\to\infty} \left(\frac{1+g}{1+i}\right)^n = 0$

$$\Rightarrow \lim_{n\to\infty} P = \frac{R}{i-g}(1-0) = \frac{R}{i-g}$$

92/ a) $N(65) = 71.8\, e^{-8.96 e^{-0.0685 \cdot 65}} \approx 65$

b) $\lim_{t\to\infty} 71.8\, e^{-8.96 e^{-0.0685t}}$

negative

$= 71.8\, e^{\left(\lim_{t\to\infty} -8.96\, e^{-0.0685t}\right)}$

$= 71.8\, e^{(-8.96 \cdot 0)} = 71.8\, e^0 = 71.8$

This is a little bigger than $N(65)$

94/ $\deg(0.17h) = 1 < 2 = \deg(h^2+2)$

$$\Rightarrow \lim_{h\to\infty} \frac{0.17h}{h^2+2} = 0$$

The concentration in blood approaches 0 over a long time.

Extra Questions

1/ $\lim_{x\to 1^+} x+2 = 1+2 = 3 > 0$

$\lim_{x\to 1^+} 1-x = 1-1 = 0$

$1-x > 0$ | 1 | $1-x < 0$ $\Rightarrow \lim_{x\to 1^+} (1-x) = 0^-$

$$\Rightarrow \lim_{x\to 1^+} \frac{x+2}{1-x} = -\infty \quad \text{(DNE)}$$

2/ $\lim_{x\to -1} x-1 = -1-1 = -2 < 0$

$\lim_{x\to -1} x^2+2x+1 = \lim_{x\to -1} (x+1)^2 = (-1+1)^2 = 0$

$(x+1)^2 > 0$ | -1 | $(x+1)^2 > 0$ $\Rightarrow \lim_{x\to -1} (x+1)^2 = 0^+$

$$\Rightarrow \lim_{x \to -1} \frac{x-1}{x^2+2x+1} = -\infty$$

3/ $\lim_{x \to 2} x^2 - 6 = 2^2 - 6 = -2 < 0$

$$\lim_{x \to 2} x^2 - 5x + 6 = \lim_{x \to 2} (x-2)(x-3) = 0$$

$x - 2 < 0$	2	$x - 2 > 0$
$x - 3 < 0$		$x - 3 < 0$
$\Downarrow$		$\Downarrow$
$(x-2)(x-3) > 0$		$(x-2)(x-3) < 0$

$\Rightarrow \lim_{x \to 2^+} x^2 - 5x + 6 = 0^-$

$\lim_{x \to 2^-} x^2 - 5x + 6 = 0^+$

$$\Rightarrow \lim_{x \to 2^+} \frac{x^2 - 6}{x^2 - 5x + 6} = \infty$$

$$\lim_{x \to 2^-} \frac{x^2 - 6}{x^2 - 5x + 6} = -\infty$$

$\Bigg\}$ $\lim_{x \to 2} \frac{x^2 - 6}{x^2 - 5x + 6}$ DNE and is neither $\pm\infty$.

§3.2 Continuity

2/ $a = -1$
a) 2, b) 2, c) 4, d) DNE, e) Limit DNE

6/ $a = 0$
a) $f(0)$ DNE, b) $-\infty$, DNE, c) $-\infty$, DNE, d) $-\infty$, DNE
e) $f(0)$ DNE <u>and</u> Limit DNE

$a = 2$
a) $f(2)$ DNE, b) -2, c) -2, d) -2, e) $f(2)$ DNE

8/ $f(x)$ is rational so is continuous anywhere it is defined.

$\Rightarrow$ Discontinuities are exactly where it is <u>not</u> defined

$(2x+1)(3x+6) = 0 \Rightarrow x = \frac{-1}{2}$ or -2

$$\lim_{x \to -\frac{1}{2}} (-2x) = -2 \cdot (-\tfrac{1}{2}) = 1 > 0$$

$$\lim_{x \to -\frac{1}{2}} (2x+1)(3x+6) = 0$$

$-\frac{1}{2}$

$2x+1<0$	$2x+1>0$
$3x+6>0$	$3x+6>0$
$\Downarrow$	$\Downarrow$
$(2x+1)(3x+6)<0$	$(2x+1)(3x+6)>0$

$$\Rightarrow \lim_{x\to -\frac{1}{2}^+} (2x+1)(3x+6) = 0^+$$

$$\lim_{x\to -\frac{1}{2}^-} (2x+1)(3x+6) = 0^-$$

$$\Rightarrow \lim_{x\to -\frac{1}{2}^+} \frac{-2x}{(2x+1)(3x+6)} = \infty$$

$$\lim_{x\to -\frac{1}{2}^-} \frac{-2x}{(2x+1)(3x+6)} = -\infty$$

$\Rightarrow \lim_{x\to -\frac{1}{2}} \frac{-2x}{(2x+1)(3x+6)}$ DNE (and is neither $\pm\infty$)

$$\lim_{x\to -2} -2x = 4 > 0$$

-2

$2x+1<0$	$2x+1<0$
$3x+6<0$	$3x+6>0$
$\Downarrow$	$\Downarrow$
$(2x+1)(3x+6)>0$	$(2x+1)(3x+6)<0$

$$\Rightarrow \lim_{x\to -2^+} (2x+1)(3x+6) = 0^-$$

$$\lim_{x\to -2^-} (2x+1)(3x+6) = 0^+$$

$$\Rightarrow \lim_{x\to -2^+} \frac{-2x}{(2x+1)(3x+6)} = -\infty$$

$$\lim_{x\to -2^-} \frac{-2x}{(2x+1)(3x+6)} = \infty$$

$\Rightarrow \lim_{x\to -2} \frac{-2x}{(2x+1)(3x+6)}$ DNE (and is neither $\pm\infty$)

14/ $$\frac{|5-x|}{x-5} = \frac{|x-5|}{x-5}$$

$$\frac{|x|}{x} = \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \end{cases} \Rightarrow \frac{|x-5|}{x-5} = \begin{cases} 1 & \text{if } x-5>0 \\ -1 & \text{if } x-5<0 \end{cases}$$

$$\Rightarrow \frac{|x-5|}{x-5} = \begin{cases} 1 & \text{if } x>5 \\ -1 & \text{if } x<5 \end{cases}$$

$\Rightarrow$ Discontinuous at $x=5$. Limit from above and below are 1 and -1 respectively. Hence limit DNE.

17/ $\ln\left|\frac{x}{x-1}\right|$ is continuous as it is a combination of continuous functions. (x, $x-1$, $|x|$, $\ln(x)$)

$\Rightarrow$ Discontinuities are precisely where it is <u>not</u> defined.

$\Rightarrow$ $x = 0$ ($\ln(0)$ DNE) or $x = 1$

$$\lim_{x\to 0}\left|\frac{x}{x-1}\right| = 0^+ \leftarrow + \text{ because of absolute value}$$

$$\lim_{u\to 0^+}\ln(u) = -\infty \quad (\text{vertical asymptote at } 0)$$

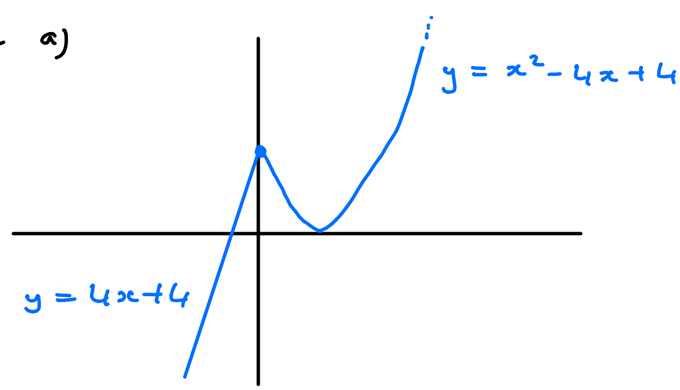
$$\Rightarrow \lim_{x\to 0}\ln\left(\left|\frac{x}{x-1}\right|\right) = -\infty \quad (\text{DNE})$$

$$\lim_{x\to 1}\left|\frac{x}{x-1}\right| = \infty \leftarrow + \text{ because of absolute value}$$

$$\lim_{u\to\infty}\ln(u) = \infty$$

$$\Rightarrow \lim_{x\to 1}\ln\left(\left|\frac{x}{x-1}\right|\right) = \infty \quad (\text{DNE})$$

23/ a)



b) There are no discontinuities

25/ kx^2, $x+k$ are polynomials so are continuous.

Thus we just need to consider $x = 2$.

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x+k = 2+k$$

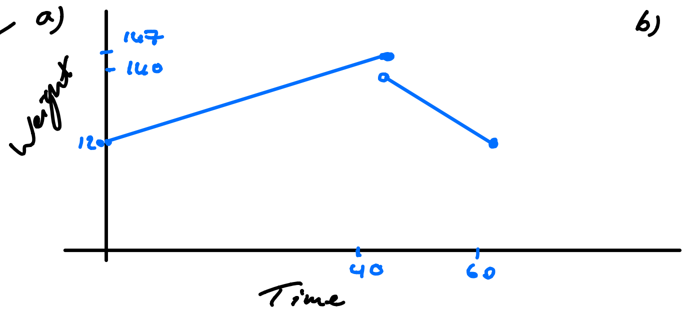
$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} kx^2 = 4k = f(2)$$

Need $2+k = 4k \Rightarrow k = 2/3$

37/ $F(x) = \begin{cases} 1.25x & \text{if } x \le 100 \\ x & \text{if } x > 100 \end{cases}$

a) $F(80) = 1.25 \times 80 = \$100$
b) $F(150) = 1 \times 150 = \$150$
c) $F(100) = 1.25 \times 100 = \$125$
d) $x = 100$.

40/ a)



b) Not continuous.
$t = 40$.

§3.3 Rates of Change

5/ Average Rate of change over $[1,4]$ $= \dfrac{\sqrt{4} - \sqrt{1}}{4-1} = \dfrac{1}{3}$

8/ Average rate of change over $[2,4]$ $= \dfrac{\ln(4) - \ln(2)}{4-2}$

15/ $\displaystyle\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{h^2 + 2h}{h} = \lim_{h \to 0} h + 2 = 2.$

17/ $\displaystyle\lim_{h \to 0} \frac{g(-1+h) - g(-1)}{h} = \lim_{h \to 0} \frac{(1 - (-1+h)^2) - (1 - (-1)^2)}{h}$

$\displaystyle = \lim_{h \to 0} \frac{\cancel{1} - \cancel{1} + 2h - h^2}{h} = \lim_{h \to 0} 2 - h = 2$

25/ a) $$\frac{P(4)-P(2)}{4-2} = 7$$

b) $$\frac{P(3)-P(2)}{3-2} = 5$$

c) $$\lim_{h\to 0} \frac{P(2+h)-P(2)}{h}$$

$$= \lim_{h\to 0} \frac{(2(2+h)^2 - 5(2+h)+6) - (2\cdot 2^2 - 5\cdot 2 + 6)}{h}$$

$$= \lim_{h\to 0} \frac{8h + 2h^2 - 5h}{h} = \lim_{h\to 0} 3 + 2h = 3$$

d) $$\lim_{h\to 0} \frac{P(4+h)-P(4)}{h}$$

$$= \lim_{h\to 0} \frac{(2(4+h)^2 - 5(4+h) + 6) - (2\cdot 4^2 - 5\cdot 4 + 6)}{h}$$

$$= \lim_{h\to 0} \frac{16h + 2h^2 - 5h}{h} = \lim_{h\to 0} 11 + 2h = 11$$

27/ a) $$\frac{N(3)-N(2)}{3-2} = -25$$

b) $$\lim_{h\to 0} \frac{N(2+h)-N(2)}{h}$$

$$= \lim_{h\to 0} \frac{(80 - 5(2+h)^2) - (80 - 5\cdot 2^2)}{h}$$

$$= \lim_{h\to 0} \frac{-20h - 5h^2}{h} = \lim_{h\to 0} -20 - 5h = -20$$

c) $$\lim_{h \to 0} \frac{N(3+h) - N(3)}{h}$$

$$= \lim_{h \to 0} \frac{(80 - 5(3+h)^2) - (80 - 5 \cdot 3^2)}{h}$$

$$= \lim_{h \to 0} \frac{-30h - 5h^2}{h} = \lim_{h \to 0} -30 - 5h = -30$$

d) Demand is decreasing. It's expected; a higher price usually lowers demand.

33/ a) $$\frac{A(15) - A(0)}{15 - 0} \approx 0.302$$

b) $$A'(15) = \lim_{h \to 0} \frac{A(15+h) - A(15)}{h}$$

$$\approx \frac{A(15.01) - A(15)}{0.01} \approx 0.356.$$

small h

34/ a) $$\frac{265 - 1042}{1960 - 1910} = -15.54$$

b) $$\frac{1043 - 265}{2010 - 1960} = 15.56$$

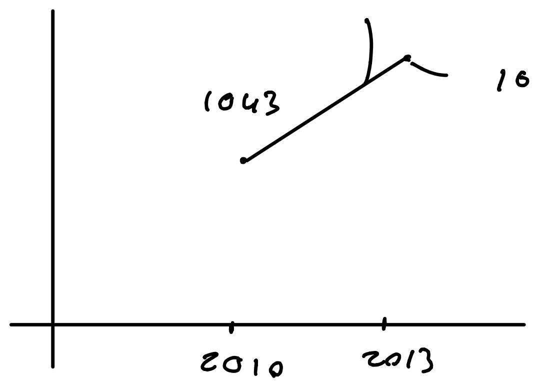
c) $$\frac{1043 - 1042}{2010 - 1910} = 0.01$$

d) $$\frac{-15.54 + 15.56}{2} = 0.01$$

No them being equal is a coincidence.

[illegible] 15.56

e)



$1043 + 3 \cdot 15 \cdot 56$

≈ 1090

This is more than actual number.