Homework 2 Solutions

§3.1 Limits

1. C

2. A

3. B

4. B and C

5. a) 3, b) 1

6. a) 4, b) 4

7. a) 0, b) DNE

8. a) 2, b) DNE

9. a) i) -1, ii) $-\frac{1}{2}$, iii) DNE, iv) DNE
   b) i), ii), iii), iv) $-\frac{1}{2}$

10. a) i), ii), iii) 1, iv) 2
    b) i), ii), iii), iv) 0

13. $\lim_{x \to 2} F(x)$ exists in Q6 because $F(x)$ approaches 4 as x approaches 2 from both sides

$\lim_{x \to -2} f(x)$ DNE in Q6 as $f(x)$ approaches -1 as x approaches -2 from below, whereas $f(x)$ approaches $-\frac{1}{2}$ as x approaches -2 from above.

Q14. $\lim_{x \to 1} f(x) = 1$ in Q10 because $f(x)$ approaches 1 as x approaches (but does not equal) 1 from above and below. $f(1) = 2$ has no relevance to $\lim_{x \to 1} f(x)$.

Q19. If you fill in table you'll see that $\frac{\sqrt{x}-2}{x-1}$ gets bigger (positively and negatively) as x approaches 1. Hence $\lim_{x \to 1} \frac{\sqrt{x}-2}{x-1}$ DNE. (and is neither ∞ nor $-\infty$)

$$\lim_{x\to 4} f(x) = 9, \quad \lim_{x\to 4} g(x) = 27 \quad \Rightarrow$$

Q23/ $$\lim_{x\to 4} \frac{f(x)}{g(x)} = \frac{9}{27} = \frac{1}{3}$$

Q28/ $$\lim_{x\to 4} (1+f(x))^2 = (1+9)^2 = 100$$

30/ $$\lim_{x\to 4} \frac{5g(x)+2}{1-f(x)} = \frac{5\cdot 27+2}{1-9} = \frac{-137}{8}$$

31/ $$\left.\begin{aligned} &\lim_{x\to 3} x^2-9 = 3^2-9 = 0 \\ &\lim_{x\to 3} x-3 = 3-3 = 0 \end{aligned}\right\} \text{UNCLEAR QUOTIENT}$$

$$\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x+3$$

$\forall\ x \neq 3$

$$\Rightarrow \lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} x+3 = 3+3 = 6.$$

34/ $$\left.\begin{aligned} &\lim_{x\to -3} x^2-9 = (-3)^2-9 = 0 \\ &\lim_{x\to -3} x^2+x-6 = (-3)^2+(-3)-6 = 0 \end{aligned}\right\} \text{UNCLEAR QUOTIENT}$$

$$\frac{x^2-9}{x^2+x-6} = \frac{(x-3)(x+3)}{(x+3)(x-2)} = \frac{x-3}{x-2}$$

$x \neq -3$

$$\lim_{x\to -3} x-3 = -3-3 = -6$$

$$\lim_{x\to -3} x-2 = -3-2 = -5$$

$$\Rightarrow \lim_{x\to -3} \frac{x^2-9}{x^2+x-6} = \lim_{x\to -3} \frac{x-3}{x-2} = \frac{-6}{-5} = \frac{6}{5}$$

38/ $$\lim_{x\to 0} \frac{\frac{-1}{(x+2)} + \frac{1}{2}}{x} = \lim_{x\to 0} \frac{\left(\frac{-2+(x+2)}{2(x+2)}\right)}{x}$$

$$= \lim_{x \to 0} \frac{x}{2(x+2)\,x} = \lim_{x \to 0} \frac{1}{2(x+2)}$$

$$\lim_{x \to 0} 1 = 1$$

$$\lim_{x \to 0} 2(x+2) = 2(0+2) = 4$$

$$\Rightarrow \lim_{x \to 0} \frac{\frac{-1}{x+2} + \frac{1}{2}}{x} = \frac{1}{4}$$

Q41 $\quad \displaystyle\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$

$$= \lim_{h \to 0} 2x + h = 2x + 0 = 2x.$$

continuous as polynomial

Q53 $\quad \displaystyle\lim_{x \to -1} f(x) = \lim_{x \to -1} x^3 + 2 = (-1)^3 + 2 = 1$

Q55 a) $\quad \displaystyle\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x - 1 = 3 - 1 = 2$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2 = 2$$

$$\Rightarrow \lim_{x \to 3} f(x) = 2$$

b) $\quad \displaystyle\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} 2 = 2$

$$\neq$$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} x + 3 = 5 + 3 = 8$$

$$\Rightarrow \lim_{x \to 5} f(x) \text{ DNE}$$

Q58 $\quad \displaystyle\lim_{x \to 3} 2x^2 + kx - 9 = 2 \cdot 3^2 + 3k - 9 = 9 + 3k$

$$\lim_{x \to 3} x^2 - 4x + 3 = 3^2 - 4 \cdot 3 + 3 = 0$$

$$9 + 3k \neq 0 \Rightarrow \lim_{x \to 3} \frac{2x^2 + kx - 9}{x^2 - 4x + 3} \text{ DNE}$$

$$9 + 3k = 0 \Rightarrow k = -3$$

$$\lim_{x \to 3} \frac{2x^2 - 3x - 9}{x^2 - 4x + 3} = \lim_{x \to 3} \frac{(2x+3)(x-3)}{(x-3)(x-1)}$$

$$= \lim_{x \to 3} \frac{2x+3}{x-1} = \frac{2 \cdot 3 + 3}{3 - 1} = \frac{9}{2}$$

$\Rightarrow k = -2$ gives limit $\frac{9}{2}$.

Q84 ✓ a) 3 million gallons , b) DNE
c) 2 million gallons , d) 16 months

Q85 ✓ a) 7.25 cents , b) 7.25 cents
c) 7.5 cents , d) DNE
e) 7.5 cents