HW1 Solutions

1.1/ Slopes and Equations of Lines

1/ $\text{Slope} = \dfrac{2-5}{-1-4} = \dfrac{-3}{-5} = \dfrac{3}{5}$

8/ $4x + 7y = 1 \Rightarrow y = \dfrac{-4}{7}x + \dfrac{1}{7} \Rightarrow \text{Slope} = \dfrac{-4}{7}$

14/ $8x = 2y - 5 \Rightarrow y = 4x + \dfrac{5}{2} \Rightarrow$ Perpendicular Slope $= \dfrac{-1}{4}$

19/ $\text{Slope} = \dfrac{3-2}{1-4} = \dfrac{-1}{3}$

Point-Slope : $y - 3 = \dfrac{-1}{3}(x-1)$

Slope-Intercept : $y = \dfrac{-1}{3}x + \dfrac{1}{3} + 3 = \dfrac{-1}{3}x + \dfrac{10}{3}$

30/ $2x - y = -4 \Rightarrow y = 2x + 4 \Rightarrow$ Parallel slope $= 2$

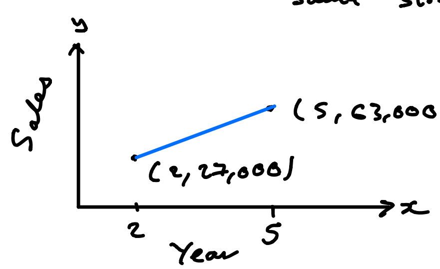
Point-Slope : $y - (-5) = 2(x-2) \Rightarrow y - 2x = -9$
$\Rightarrow 2x + (-1)y = 9$

35/ Slope of line through $(4,3), (2,0) = \dfrac{0-3}{2-4} = \dfrac{-3}{-2} = \dfrac{3}{2}$

Slope of line through $(2,0), (-18,-12) = \dfrac{-12-0}{-18-2} = \dfrac{-12}{-20} = \dfrac{3}{5}$

$\dfrac{3}{2} \neq \dfrac{3}{5} \Rightarrow (4,3), (2,0), (-18,-12)$ do not lie on same straight line

61/ a)

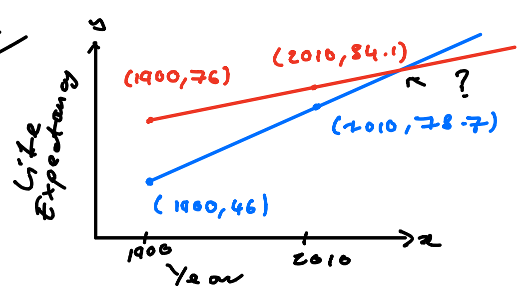


$\text{Slope} = \dfrac{63000 - 27000}{5 - 2} = \dfrac{36000}{3} = 12000$

$\Rightarrow y - 27000 = 12000(x-2) \Rightarrow y = 12000x + 3000$

b) $100{,}000 = 12000x + 3000 \Rightarrow x = \dfrac{97000}{12000}$ years

69/



$y - 76 = \dfrac{84.1 - 76}{2010 - 1900}(x - 1900)$

$\Rightarrow y = \dfrac{8.1}{110}(x - 1900) + 76$

$y - 46 = \dfrac{78.7 - 46}{2010 - 1900}(x - 1900)$

$\Rightarrow y = \dfrac{32.7}{110}(x - 1900) + 46$

$$\frac{8.1}{110}(x-1900)+76 = \frac{32.7}{110}(x-1900)+46$$

$$\Rightarrow (x-1900) = \frac{30}{\left(\frac{32.7-8.1}{110}\right)} = \frac{3300}{24.6}$$

$$\Rightarrow y = \frac{8.1}{110}\cdot\frac{3300}{24.6} + 76 = 85.9 \text{ years.}$$

1.2/ Linear Functions and Applications

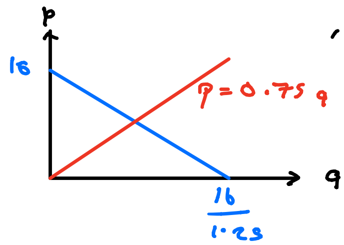
27/ a) $p = \$16$, b) $p = 16 - 1.25\times 4 = \$11$, c) $p = \$6$

d) $8 = 16 - 1.25q \Rightarrow q = \frac{8}{1.25} = 6.4 \Rightarrow$ 640 watches

e) 480 watches,

f) 320 watches

g)



, h) 0 watches, i) 1333 watches,

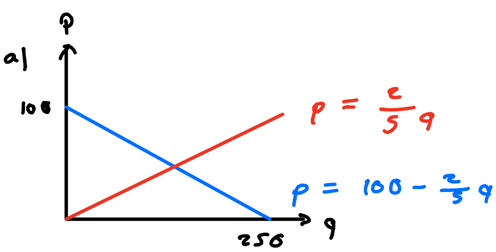
j) 2667 watches

k) $0.75q = 16 - 1.25q \Rightarrow q = \frac{16}{2} = 8$

$\Rightarrow p = 6$

$\Rightarrow$ Equilibrium is when $p = \$6$ and 800 watches are supplied/sold.

29/ a)

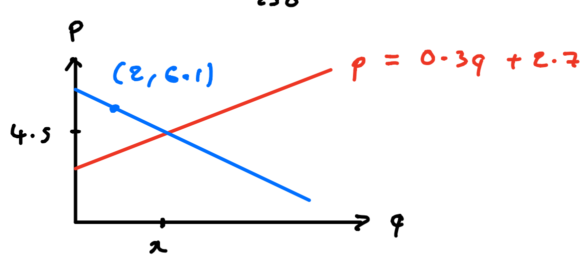


b) $100 - \frac{2}{5}q = \frac{2}{5}q$

$\Rightarrow \frac{4}{5}q = 100 \Rightarrow q = \frac{500}{4} = 125$

$\Rightarrow p = \$50$

31/



$0.3x + 2.7 = 4.5$

$\Rightarrow x = \frac{1.8}{0.3} = 6$

Point-Slope : $p - 4.5 = \frac{4.5 - 6.1}{6 - 2}(q - 6)$

$\Rightarrow p = -0.4q + 6.9 = D(q)$

33/ a) $R(x) = C(x) \Rightarrow 15x = 5x + 20 \Rightarrow 10x = 20 \Rightarrow x = 2$

b) $R(100) - C(100) = 1500 - 520 = \$980$

c) $500 = 10x - 20 \Rightarrow x = 52$

56/ a) $C(x) = 1140x + 486000$, b) $C(500) = 1140\times 500 + 486000 = \$1056000$

c) $1000000 = 1140x + 486000 \Rightarrow x = 450.9.$

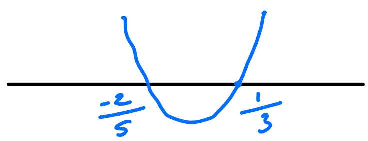
2.1/ Properties of Functions

4/ Not a function $f(1) = 1$ and -1

7/ $|-1| = |1| = 1$ $\Rightarrow$ $x = 1$ has 2 possible y, 1 and -1.
$\Rightarrow$ Not a function

8/ $5 = (1)^2 + 4 = (-1)^2 + 4$ $\Rightarrow$ $x = 5$ has 2 possible y, 1 and -1
$\Rightarrow$ Not a function.

30/ Domain of $f(x)$ = x such that $15x^2 + x - 2 \geq 0$
$\qquad = (5x+2)(3x-1)$



$\Rightarrow$ Domain $= (-\infty, -\frac{2}{5}] \cup [\frac{1}{3}, \infty)$

32/ Domain of $f(x)$ = x such that $\dfrac{x^2}{3-x} \geq 0$

$\Leftrightarrow$ $3 - x > 0$ $\Leftrightarrow$ $3 > x$

$\Rightarrow$ Domain $= (-\infty, 3)$

35/ Domain $= (-\infty, \infty)$
Range $= (-\infty, 12]$

38/ Domain $= [-2, 4]$
Range $= [0, 5]$

55/ a) $f(x+h) = \dfrac{1}{x+h}$, b) $f(x+h) - f(x) = \dfrac{1}{x+h} - \dfrac{1}{x}$

c) $$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{-1}{x(x+h)}$$

$$\frac{x - (x+h)}{x(x+h)} = \frac{-h}{x(x+h)}$$

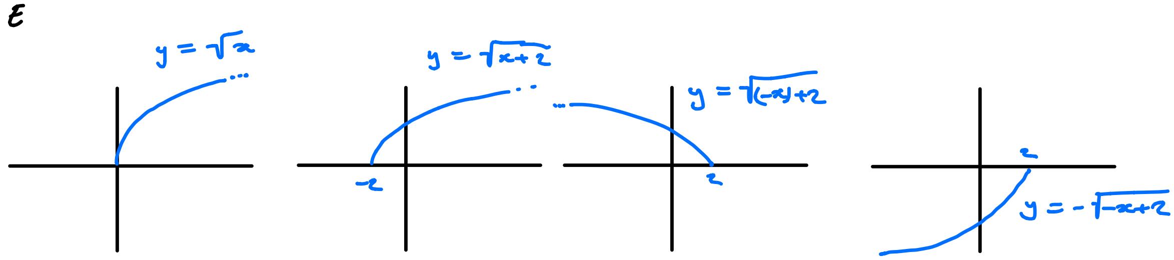
71/ a) No, b) Year t, c) Price of Silver, $S(t)$, d) $[2000, 2013]$,
e) $[4, 35]$, f) \$15, g) 2011

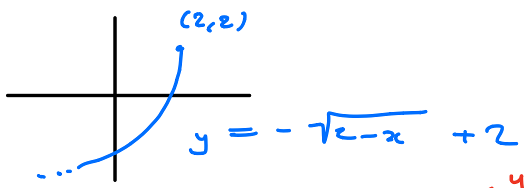
77/ a) ≈ 140 m, b) ≈ 250 m

2.2/ Quadratic Functions; Translations and Reflections

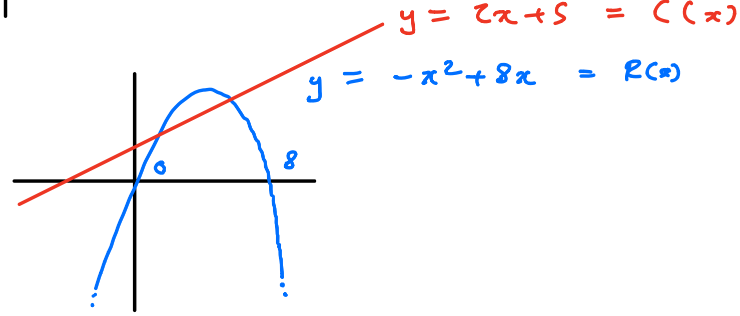
5/ A
8/ E

38/





49/ a)



b) $R(x) = C(x) \Rightarrow -x^2+8x = 2x+5 \Rightarrow x^2-6x+5=0$

$\Rightarrow (x-5)(x-1) = 0 \Rightarrow x = 1$ or $5 \Rightarrow x = 1$

is min break-even quantity

c) $R(4) = 16 \Rightarrow$ \$16000 is max revenue

d) $P(x) = -x^2+6x-5 \Rightarrow$ Max profit is $P\left(\frac{-6}{-2}\right) = 4$

$\Rightarrow$ Max profit is \$4000.

62/ Max rate $= f\left(\frac{-0.49}{2\times(-0.0335)}\right)$

Increasing for $t < \frac{-0.49}{2\times(-0.0335)}$

Decreasing for $t > \frac{-0.49}{2\times(-0.0335)}$

2.4/ <u>Exponential Functions</u>

5/ $y = \left(\frac{1}{3}\right)^{1-x} = y = (3^{-1})^{1-x} = 3^{x-1} = \frac{1}{3}\cdot 3^x \Rightarrow C$

9/ A

24/ $2^{x^2-4x} = \left(\frac{1}{16}\right)^{x-4} = (2^{-4})^{x-4} = 2^{4-x}$

$\Rightarrow x^2-4x = 4-x \Rightarrow x^2-3x-4 = 0$

$\Rightarrow (x-4)(x+1) = 0 \Rightarrow x = -1$ or 4

40/a) $5000(1+r)^5 = 6100 \Rightarrow r = \sqrt[5]{\frac{6100}{5000}} - 1$

b) $5000\left(1+\frac{r}{4}\right)^{20} = 6100 \Rightarrow r = 4\sqrt[20]{\frac{6100}{5000}} - 4$

48/ a) $f(1) = 500 \cdot 2^3 = 4000$

b) $f(0) = 500 \cdot 2^0 = 500$

c) $f(t) = 1000 \Rightarrow 500\, 2^{3t} = 1000 \Rightarrow 2^{3t} = 2 = 2^1$

$\Rightarrow 3t = 1 \Rightarrow t = \frac{1}{3}$.

d) $32000 = 500\, 2^{3t} \Rightarrow 2^{3t} = 64 = 2^6 \Rightarrow t = 2$.

2.5/ <u>Logarithmic Functions</u>

9/ $\ln\left(\frac{1}{e}\right) = -1 \Rightarrow \frac{1}{e} = e^{-1}$

20/ $\log_8\left(\sqrt[4]{\frac{1}{2}}\right) = \log_8\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\right) = \frac{1}{4}\log_8\left(\frac{1}{2}\right) = \frac{-1}{4}\log_8(2)$

$2^3 = 8 \Rightarrow 8^{\frac{1}{3}} = 2 \Rightarrow \log_8(2) = \frac{1}{3}$

$\Rightarrow \log_8\left(\sqrt[4]{\frac{1}{2}}\right) = \frac{-1}{4} \cdot \frac{1}{3} = \frac{-1}{12}$.

32/ $\ln\left(\frac{9\sqrt[3]{5}}{\sqrt[4]{3}}\right) = \ln(9) + \ln(\sqrt[3]{5}) - \ln(\sqrt[4]{3})$

$= 2\ln(3) + \frac{1}{3}\ln(5) - \frac{1}{4}\ln(3)$

$= \frac{7}{4}\ln(3) + \frac{1}{3}\ln(5)$

55/ $\ln(x) + \ln(3x) = -1 \Rightarrow \ln(3x^2) = -1$

$\Rightarrow 3x^2 = e^{-1} \Rightarrow x = \pm\sqrt{\frac{e^{-1}}{3}}$

72/ $(\log(x+2))^2 \neq \log((x+2)^2) = 2\log(x+2)$

$\log(x+2) \neq \log(x) + \log(2)$

$\log(2) \neq 100$

77/ $f(t) = 600\, e^{rt}$

$f(14) = 600\, e^{14r} = 1240$

$\Rightarrow r = \frac{1}{14}\ln\left(\frac{1240}{600}\right)$.