Homework 12 Solutions

§7.4 / 3, 7, 9, 11, 21, 24, 27, 36, 44, 47, 51, 58, 59, 60

§7.5 / 3, 9, 15, 16, 18, 21, 33, 34

7.4 /

3/ $\displaystyle\int_{-1}^{2} 5t - 3\, dt = \frac{5}{2}t^2 - 3t \Big|_{-1}^{2} = \left(\frac{5}{2}2^2 - 3\cdot 2\right) - \left(\frac{5}{2}(-1)^2 - 3(-1)\right)$

$= \dfrac{-3}{2}$

7/ $v = 4u + 1 \Rightarrow \dfrac{dv}{du} = 4 \Rightarrow du = \dfrac{dv}{4}$

$\displaystyle\Rightarrow \int 3\sqrt{4u+1}\, du = \int \frac{3}{4}\sqrt{v}\, dv = \frac{3}{4}\cdot\frac{2}{3}\cdot v^{3/2} + C$

$= \frac{1}{2}(4u+1)^{3/2} + C$

$\displaystyle\Rightarrow \int_0^2 3\sqrt{4u+1}\, du = \frac{1}{2}(4u+1)^{3/2}\Big|_0^2 = \frac{1}{2}(9)^{3/2} - \frac{1}{2}(1)^{3/2} = 13$

9/ $\displaystyle\int_0^4 2t^{1/2} - 2t\, dt = 2\frac{2}{3}\cdot t^{3/2} - t^2\Big|_0^4 = \frac{4}{3}4^{3/2} - 16 = \frac{-16}{3}$

11/ $\displaystyle\int_1^4 5y^{3/2} + 3y^{1/2}\, dy = 5\cdot\frac{2}{5}y^{5/2} + 3\cdot\frac{2}{3}y^{3/2}\Big|_1^4$

$= (2\cdot 4^{5/2} + 2\cdot 4^{3/2}) - (2\cdot 1^{5/2} + 2\cdot 1^{3/2})$

$= 76$

21/ $u = 2y^2 - 3 \Rightarrow \dfrac{du}{dy} = 4y \Rightarrow dy = \dfrac{du}{4y}$

$\displaystyle\Rightarrow \int y(2y^2-3)^5\, dy = \int \frac{1}{4}u^5\, du = \frac{1}{24}u^6 + C = \frac{1}{24}(2y^2-3)^6 + C$

$\displaystyle\Rightarrow \int_{-1}^{0} y(2y^2-3)^5\, dy = \frac{1}{24}(2y^2-3)^6\Big|_{-1}^{0} = \left(\frac{1}{24}(-3)^6\right) - \left(\frac{1}{24}(-1)^6\right)$

$= \dfrac{91}{3}$

24/ $\displaystyle\int_1^8 3y^{-2/3} - y^{-\frac{1}{3}}\,dy = 3\cdot 3y^{\frac{1}{3}} - \frac{3}{2}y^{\frac{2}{3}}\Big|_1^8$

$$= \left(9\cdot 8^{1/3} - \frac{3}{2}\,8^{2/3}\right) - \left(9 - \frac{3}{2}\right)$$

$$= \frac{9}{2}$$

27/ $u = x^{4/3} + 9 \Rightarrow \dfrac{du}{dx} = \dfrac{4}{3}x^{1/3} \Rightarrow dx = \dfrac{3}{4}\cdot\dfrac{du}{x^{1/3}}$

$$\Rightarrow \int x^{1/3}\sqrt{x^{4/3}+9}\,dx = \int \frac{3}{4}\sqrt{u}\,du = \frac{3}{4}\cdot\frac{2}{3}\cdot u^{3/2} + C$$

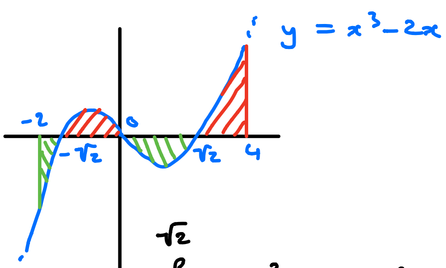
$$= \frac{1}{2}\left(x^{4/3}+9\right)^{3/2} + C$$

$$\Rightarrow \int_0^8 x^{1/3}\sqrt{x^{4/3}+9}\,dx = \frac{1}{2}\left(x^{4/3}+9\right)^{3/2}\Big|_0^8 = \frac{1}{2}\left(8^{4/3}+9\right)^{3/2} - \frac{1}{2}\,9^{3/2}$$

$$= 49$$

36/ $x^3 - 2x = x(x^2-2)$

$\Rightarrow$ $y = x^3 - 2x$ crosses x-axis at 0, $\sqrt{2}$ and $-\sqrt{2}$



$$\int_{\sqrt{2}}^4 x^3 - 2x\,dx = \frac{1}{4}x^4 - x^2\Big|_{\sqrt{2}}^4$$

$$= \left(\frac{4^4}{4} - 4^2\right) - \left(\frac{1}{4}\cdot 4 - 2\right)$$

$$\int_0^{\sqrt{2}} 2x - x^3\,dx = x^2 - \frac{1}{4}x^4\Big|_0^{\sqrt{2}} = 1 \qquad = 64 - 16 + 1 = 49$$

$$\int_{-\sqrt{2}}^0 x^3 - 2x\,dx = \frac{1}{4}x^4 - x^2\Big|_{-\sqrt{2}}^0 = 0 - \left(\frac{1}{4}\cdot 4 - 2\right) = 1$$

$$\int_{-2}^{-\sqrt{2}} 2x - x^3\,dx = x^2 - \frac{1}{4}x^4\Big|_{-2}^{-\sqrt{2}} = (2-1) - (4-4) = 1$$

$\Rightarrow$ Total area enclosed $= 49 + 1 + 1 + 1 = 52$

44/ $u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

$$\Rightarrow \int \frac{\ln(x)}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln(x))^2 + C$$

$$\Rightarrow \int_1^e \frac{\ln(x)}{x}\,dx = \frac{1}{2}(\ln(x))^2 \Big|_1^e = \frac{1}{2}$$

$$-\int_{e^{-1}}^1 \frac{\ln(x)}{x}\,dx = -\frac{1}{2}(\ln(x))^2 \Big|_{e^{-1}}^1 = \frac{1}{2}$$

$\Rightarrow$ Shaded Region has area $\frac{1}{2} + \frac{1}{2} = 1$.

47/ Area of region above x-axis $= 2\times 1 + \frac{2\times 2}{2} + \frac{\pi \cdot 3^2}{4} = 4 + \frac{9}{4}\pi$

Area of region below x-axis $= \frac{\pi \cdot 3^2}{4} + \frac{3\times 8}{2} = 12 + \frac{9}{4}\pi$

$$\Rightarrow \int_0^{16} f(x)\,dx = \left(4 + \frac{9}{4}\pi\right) - \left(12 + \frac{9}{4}\pi\right) = -8$$

51/ $$\int_{-1}^0 2x + 3\,dx = x^2 + 3x \Big|_{-1}^0 = 0 - \left((-1)^2 + 3(-1)\right) = 2$$

$$\int_0^4 -\frac{x}{4} - 3\,dx = \frac{-x^2}{8} - 3x \Big|_0^4 = \frac{-16}{8} - 12 = -14$$

$$\Rightarrow \int_{-1}^4 f(x)\,dx = 2 + (-14) = -12$$

58/ $u = \ln(t+1) \Rightarrow \frac{du}{dt} = \frac{1}{t+1} \Rightarrow dt = (t+1)\,du$

$$\Rightarrow \int \frac{80\ln(t+1)}{t+1}\,dt = \int 80u\,du = 40u^2 + C = 40(\ln(t+1))^2 + C$$

a) $\displaystyle\int_0^{24} \frac{80\ln(t+1)}{t+1}\,dt = 40(\ln(t+1))^2\Big|_0^{24} = 40(\ln(25))^2 \simeq 414$

b) $\displaystyle\int_{24}^{48} \frac{80\ln(t+1)}{t+1}\,dt = 40(\ln(t+1))^2\Big|_{24}^{48} = 40(\ln(49))^2 - 40(\ln(25))^2$

$\simeq 191$

c) In the long run the number of barrels will decrease to 0.

59/ a) $\displaystyle\int_1^{2} 0.6 + 4(t+1)^{-3}\,dt = 0.6t - 2(t+1)^{-2}\Big|_1^{2}$

$= ((0.6)\cdot 2 - 2(3)^{-2}) - ((0.6) - 2\cdot 2^{-2})$

≈ 0.8778

b) $\displaystyle\int_2^{3} 0.6 + 4(t+1)^{-3}\,dt = 0.6t - 2(t+1)^{-2}\Big|_2^{3}$

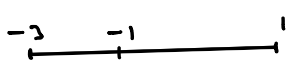
$= ((0.6)\cdot 3 - 2(4)^{-2}) - ((0.6)\cdot 2 - 2\cdot 3^{-2})$

≈ 0.6972

60/ $\displaystyle\int_0^{3.5} 150e^{0.2x}\,dx = \frac{150}{0.2}e^{0.2x}\Big|_0^{3.5} = \frac{150}{0.2}(e^{7} - 1) \approx 760.3$

7.5/

3/ Intersections : $x^3 + 1 = 0 \Rightarrow x = -1$



$\displaystyle\int_{-3}^{-1} (x^3+1) - 0\,dx = \frac{1}{4}x^4 + x\Big|_{-3}^{-1} = \left(\frac{1}{4} - 1\right) - \left(\frac{81}{4} - 3\right)$

$= \frac{-3}{4} - \frac{81}{4} + \frac{12}{4} = \frac{-72}{4} = -18$

Negative so correct area is 18.

$$\int_{-1}^{1} x^3+1 \; dx = \frac{1}{4}x^4 + x \Big|_{-1}^{1} = 2$$

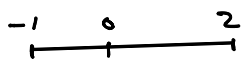
$\Rightarrow$ Area enclosed is $18 + 2 = 20$

9/ Intersections : $x^2 = 2x \Rightarrow x(x-2) = 0 \Rightarrow x = 0, 2$

$$\int_{0}^{2} x^2 - 2x \; dx = \frac{1}{3}x^3 - x^2 \Big|_{0}^{2} = \frac{8}{3} - 4 = \frac{-4}{3}$$

Negative so area enclosed is $\frac{4}{3}$.

15/ Intersections : $2e^{2x} = e^{2x} + 1 \Rightarrow e^{2x} = 1 \Rightarrow 2x = 0$
$\Rightarrow x = 0$



$$\int_{-1}^{0} \left(2e^{2x} - (e^{2x}+1)\right) dx = \int_{-1}^{0} e^{2x} - 1 \; dx = \frac{1}{2}e^{2x} - x \Big|_{-1}^{0}$$

$$= \frac{1}{2} - \left(\frac{1}{2}e^{-2} + 1\right)$$

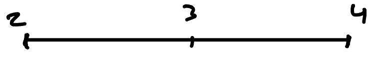
$$= \frac{-1}{2} - \frac{1}{2}e^{-2}$$

Negative so area is $\frac{1}{2} + \frac{1}{2}e^{-2}$

$$\int_{0}^{2} (2e^{2x}) - (e^{2x}+1) \; dx = \frac{1}{2}e^{2x} - x \Big|_{0}^{2} = \frac{1}{2}e^4 - 2 - \frac{1}{2} > 0$$

$\Rightarrow$ Area enclosed $= \frac{1}{2} + \frac{1}{2}e^{-2} + \frac{1}{2}e^4 - \frac{5}{2} = \frac{1}{2}(e^{-2} + e^4) - 2$

16/ Intersections : $\frac{x-1}{4} = \frac{1}{x-1} \Rightarrow (x-1)^2 = 4 \Rightarrow x - 1 = \pm 2$
$\Rightarrow x = -1$ or 3



$$\int_{2}^{3} \frac{x-1}{4} - \frac{1}{x-1} = \frac{(x-1)^2}{8} - \ln|x-1| \Big|_{2}^{3} = \left(\frac{2^2}{8} - \ln(2)\right) - \frac{1}{8}$$

$$= \frac{3}{8} - \ln(2) < 0$$

$\Rightarrow$ Area is $\ln(2) - \frac{3}{8}$

$$\int_2^4 \frac{x-1}{4} - \frac{1}{x-1} \, dx = \frac{(x-1)^2}{8} - \ln|x-1| \Big|_2^4 = \left(\frac{3^2}{8} - \ln(3)\right) - \left(\frac{2^2}{8} - \ln(2)\right)$$

$$= \frac{5}{8} + \ln(2) - \ln(3) > 0$$

$\Rightarrow$ Area enclosed $= \ln(2) - \frac{3}{8} + \frac{5}{8} + \ln(2) - \ln(3)$

$$= 2\ln(2) - \ln(3) + \frac{1}{4}$$

18/ Intersections : $2x^3 + x^2 + x + 5 = x^3 + x^2 + 2x + 5$

$\Rightarrow x^3 - x = 0 \Rightarrow x = 0, 1, -1$

$$\int_{-1}^0 x^3 - x \, dx = \frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_{-1}^0 = 0 - \left(\frac{1}{4} - \frac{1}{2}\right) = \frac{1}{4}$$

$$\int_0^1 x^3 - x \, dx = \frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_0^1 = -\frac{1}{4}$$

Negative so area is $\frac{1}{4}$.

$\Rightarrow$ Total area enclosed is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

21/ Intersections : $x^{4/3} = 2x^{1/3} \Rightarrow x^{4/3} - 2x^{1/3} = 0$

$\Rightarrow x^{1/3}(x-2) = 0 \Rightarrow x = 0$ or 2

$$\int_0^2 x^{4/3} - 2x^{1/3} \, dx = \frac{3}{7}x^{7/3} - 2 \cdot \frac{3}{4}x^{4/3} \Big|_0^2$$

$$= \frac{3}{7} \cdot 2^{7/3} - \frac{3}{2} 2^{4/3} < 0$$

$\Rightarrow$ Area enclosed $= \frac{3}{2} 2^{4/3} - \frac{3}{7} 2^{7/3}$

33/ $$\int \frac{200}{(3q+1)^2} \, dq = \frac{200}{3} \cdot \frac{-1}{(3q+1)} + C$$

$$\Rightarrow \int_0^3 \frac{200}{(3q+1)^2}\,dq = \frac{-200}{3(3q+1)}\Bigg|_0^3 = \frac{-200}{30} - \left(\frac{-200}{3}\right)$$

$$= \frac{1800}{30} = 60$$

$$D(3) = \frac{200}{10^2} = 2 \Rightarrow \text{Consumer Surplus} = 60 - 2\times 3 = 54.$$

34

$$\int \frac{32000}{(2q+8)^3}\,dq = 32000 \cdot \frac{1}{2} \cdot \frac{1}{-2} \cdot \frac{1}{(2q+8)^2} + C$$

$$= \frac{-8000}{(2q+8)^2} + C$$

$$\Rightarrow \int_0^6 \frac{32000}{(2q+8)^3}\,dq = \frac{-8000}{(2q+8)^2}\Bigg|_0^6 = \frac{-8000}{20^2} - \frac{-8000}{8^2}$$

$$D(6) = \frac{32000}{20^3} = 4 \Rightarrow \text{Consumer Surplus} = \frac{8000}{8^2} - \frac{8000}{400} - 4\times 6$$

$$= 125 - 20 - 24 = 81$$