Homework 11 Solutions

§7.2 3, 8, 9, 15, 19, 24, 25, 30, 31, 32, 36, 38, 39, 41, 45, 46

§7.3 1, 15, 16, 17, 18, 19

7.2

3/ $u = 2x+3 \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$

$\Rightarrow \int 4(2x+3)^4\,dx = \int 4(2x+3)^4 \cdot \frac{du}{2} = \int 2u^4\,du$

$= \frac{2}{5}u^5 + C = \frac{2}{5}(2x+3)^5 + C$

8/ $u = 2x^3+7 \Rightarrow \frac{du}{dx} = 6x^2 \Rightarrow dx = \frac{du}{6x^2}$

$\Rightarrow \int \frac{6x^2}{(2x^3+7)^{3/2}}\,dx = \int \frac{6x^2}{(2x^3+7)^{3/2}} \cdot \frac{du}{6x^2} = \int u^{-3/2}\,du$

$= -2u^{-\frac{1}{2}} + C = -2(2x^3+7)^{-\frac{1}{2}} + C$

9/ $u = 4z^2-5 \Rightarrow \frac{du}{dz} = 8z \Rightarrow dz = \frac{du}{8z}$

$\Rightarrow \int z\sqrt{4z^2-5}\,dz = \int z\sqrt{4z^2-5} \cdot \frac{du}{8z} = \int \frac{1}{8}\sqrt{u}\,du$

$= \frac{1}{8}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{12}(4z^2-5)^{3/2} + C$

15/ $u = 1/z \Rightarrow \frac{du}{dz} = \frac{-1}{z^2} \Rightarrow dz = -z^2\,du$

$\Rightarrow \int \frac{e^{1/z}}{z^2}\,dz = \int \frac{e^{1/z}}{z^2} \cdot -z^2\,du = \int -e^u\,du = -e^u + C$

$= -e^{\frac{1}{z}} + C$

19/ $u = x^4+4x^2+7 \Rightarrow \frac{du}{dx} = 4x^3+8x \Rightarrow dx = \frac{du}{4x^3+8x}$

$\Rightarrow \int \frac{x^3+2x}{x^4+4x^2+7}\,dx = \int \frac{1}{4}\cdot\frac{1}{u}\,du = \frac{1}{4}\ln|u| + C = \frac{1}{4}\ln|x^4+4x^2+7| + C$

24/ $u = 8 - r \Rightarrow \frac{du}{dr} = -1 \Rightarrow dr = -du$

$(r = 8-u)$

$$\Rightarrow \int 4r\sqrt{8-r}\, dr = \int 4r\sqrt{8-r} \cdot -du = \int -4(8-u)\sqrt{u}\, du$$

$$= \int -32u^{1/2} + 4u^{3/2}\, du = -32 \cdot \frac{2}{3} u^{3/2} + 4 \cdot \frac{2}{5} u^{5/2} + C$$

$$= -\frac{64}{3}(8-r)^{3/2} + \frac{8}{5}(8-r)^{5/2} + C$$

25/ $v = u - 1 \Rightarrow \frac{du}{dv} = 1 \Rightarrow du = dv$

$(u = v+1)$

$$\Rightarrow \int \frac{u}{\sqrt{u-1}}\, du = \int \frac{u}{\sqrt{u-1}}\, dv = \int \frac{v+1}{\sqrt{v}}\, dv = \int v^{\frac{1}{2}} + v^{-\frac{1}{2}} dv$$

$$= \frac{2}{3} v^{3/2} + 2v^{\frac{1}{2}} + C = \frac{2}{3}(u-1)^{3/2} + 2(u-1)^{1/2} + C$$

30/ $u = 2 + \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

$$\Rightarrow \int \frac{\sqrt{2+\ln(x)}}{x}\, dx = \int \frac{\sqrt{2+\ln(x)}}{x} \cdot x\,du = \int \sqrt{u}\, du$$

$$= \frac{2}{3} u^{3/2} + C = \frac{2}{3}(2+\ln(x))^{3/2} + C$$

31/ $u = e^{2x} + 5 \Rightarrow \frac{du}{dx} = 2e^{2x} \Rightarrow dx = \frac{du}{2e^{2x}}$

$$\Rightarrow \int \frac{e^{2x}}{e^{2x}+5}\, dx = \int \frac{e^{2x}}{e^{2x}+5} \cdot \frac{du}{2e^{2x}} = \int \frac{1}{2} \frac{1}{u}\, du$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|e^{2x}+5| + C$$

32/ $u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

$$\Rightarrow \int \frac{1}{x\ln(x)}\, dx = \int \frac{1}{x\ln(x)} \cdot x\,du = \int \frac{1}{u}\, du = \ln|u| + C$$

$$= \ln|\ln(x)| + C$$

36/ $u = 5\sqrt{x} + 2 \Rightarrow \frac{du}{dx} = \frac{5}{2} x^{-\frac{1}{2}} \Rightarrow dx = \frac{2}{5}\sqrt{x}\, du$

$$\Rightarrow \int \frac{10^{(5\sqrt{x}+2)}}{\sqrt{x}}\, dx = \int \frac{10^{(5\sqrt{x}+2)}}{\sqrt{x}} \cdot \frac{2}{5}\sqrt{x}\, du$$

$$= \int \frac{2}{5} 10^{u}\, du = \frac{2}{5} \frac{1}{\ln(10)} 10^{u} + C = \frac{2}{5} \cdot \frac{1}{\ln(10)} 10^{(5\sqrt{x}+2)} + C$$

38/ $$\frac{(x^2+2)^2}{2} \underbrace{+C}_{\text{any constant}} = \frac{x^2+4x+4}{2} + C = \frac{x^2}{2} + 2x \underbrace{+(2+C)}_{\text{any constant}}$$

39/ a) $u = x^2 + 27{,}000 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$$\Rightarrow \int 4x(x^2+27000)^{-2/3}\, dx = \int 4x(x^2+27000)^{-2/3} \frac{du}{2x}$$

$$= \int 2u^{-2/3}\, du = 6u^{1/3} + C = 6(x^2+27000)^{1/3} + C$$

$R(125) = 29.591$

$$\Rightarrow 6((125)^2+27000)^{1/3} + C = 29.591 \Rightarrow C \approx -180$$

$$\Rightarrow R(x) = 6(x^2+27000)^{1/3} - 180$$

b) $R(x) \geq 40 \Rightarrow 6(x^2+27000)^{1/3} - 180 \geq 40$

$$\Rightarrow (x^2+27000)^{1/3} \geq \frac{220}{6}$$

$$\Rightarrow x^2+27000 \geq \left(\frac{220}{6}\right)^3$$

$$\Rightarrow x \geq \sqrt{\left(\frac{220}{6}\right)^3 - 27000} \approx 150$$

41/ a) $u = 5x^2 + e \Rightarrow \frac{du}{dx} = 10x \Rightarrow dx = \frac{du}{10x}$

$\Rightarrow \int \frac{60x}{5x^2+e}\, dx = \int \frac{60x}{5x^2+e} \cdot \frac{du}{10x} = \int 6 \cdot \frac{1}{u}\, du$

$= 6 \cdot \ln|u| + C = 6 \cdot \ln|5x^2+e| + C$

$C(0) = 10$

$\Rightarrow 6 \cdot \ln|e| + C = 10 \Rightarrow C = 4$

$\Rightarrow C(x) = 6 \cdot \ln|5x^2+e| + 4$

b) $C(5) = 6 \cdot \ln|125+e| + 4 > 20 \Rightarrow$ Should seek new source

45/

a) $u = -\frac{t^2}{5} \Rightarrow \frac{du}{dt} = \frac{-2t}{5} \Rightarrow dt = \frac{-5}{2t} du$

$\Rightarrow \int 500t\, e^{-\frac{t^2}{5}}\, dt = \int 500t\, e^{-\frac{t^2}{5}} \cdot \frac{-5}{2t}\, du$

$= \int -1250 e^u\, du = -1250\, e^u + C = -1250 e^{-\frac{t^2}{5}} + C$

$P(0) = 2000 \Rightarrow -1250 + C = 2000 \Rightarrow C = 3250$

$\Rightarrow P(x) = 3250 - 1250 e^{-\frac{t^2}{5}}$

b) $P(3) \approx 3043$

46/ $u = t^2+2 \Rightarrow \frac{du}{dt} = 2t \Rightarrow dt = \frac{du}{2t}$

$\Rightarrow \int \frac{100t}{t^2+2}\, dt = \int 50 \cdot \frac{1}{u}\, du = 50 \ln|u| + C$

$$= 50 \ln|t^2+2| + C$$

$$N(0) = 37 \Rightarrow 50\ln(2) + C = 37 \Rightarrow C \approx 2.343$$

$$\Rightarrow N(t) = 50\ln|t^2+2| + 2.343$$

b) $N(21) \approx 307$

§7.3

1/ An indefinite integral is a <u>general antiderivative</u>.

A definite integral is a <u>net area</u>

15/ a) $\displaystyle\int_0^4 f(x)\,dx$ = Area of triangle $= \dfrac{4\times 2}{2} = 4$

b) $\displaystyle\int_0^4 f(x)\,dx$ = Sum of Areas of triangles $= \dfrac{3\times 3}{2} + \dfrac{1\times 1}{2} = 5$

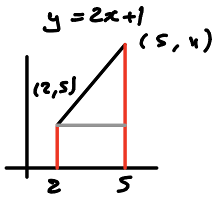
16/ a) $\displaystyle\int_0^6 f(x)\,dx$ = Area of rectangle + Area of quarter circle $= 4\times 2 + \dfrac{\pi \cdot 4^2}{4} = 8 + 4\pi$

b) $\displaystyle\int_0^6 f(x)\,dx$ = Area of triangle + Area of quarter circle $= \dfrac{4\times 2}{2} + \dfrac{\pi \cdot 2^2}{4} = 4 + \pi$

17/ $\displaystyle\int_{-4}^0 \sqrt{16-x^2}\,dx$ = Area of quarter circle radius 4 $= \dfrac{\pi \cdot 4^2}{4} = 4\pi$

18/ $\displaystyle\int_{-3}^3 \sqrt{9-x^2}\,dx$ = Area of half circle radius 3 $= \dfrac{\pi \cdot 3^2}{2} = \dfrac{9\pi}{2}$

19/



$$\Rightarrow \int_2^5 (1+2x)\,dx = 3\times 5 + \frac{3\times 6}{2} = 24$$