Homework 10 Solutions

We'll repeatedly use the fact: $\frac{d}{dx}(g(y)) = g'(y)\frac{dy}{dx}$

§6.4 1, 7, 10, 15, 16, 19, 23, 27, 38, 45

§7.1 5, 7, 14, 23, 36, 42, 43, 49, 55, 58

6.4

1/ $\frac{d}{dx}(6x^2+5y^2) = \frac{d}{dx}(36)$

$\Rightarrow 12x + 10y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-6x}{5y}$

7/ $\frac{d}{dx}(3x^2) = \frac{d}{dx}\left(\frac{2-y}{2+y}\right)$

$\Rightarrow 6x = \frac{(-1)(2+y) - (2-y)\cdot 1}{(2+y)^2}\cdot\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-3x(2+y)^2}{2}$

10/ $\frac{d}{dx}(4x^{\frac{1}{2}} - 8y^{\frac{1}{2}}) = \frac{d}{dx}(6y^{3/2})$

$= 2x^{-\frac{1}{2}} - 4y^{-\frac{1}{2}}\frac{dy}{dx} = 9y^{\frac{1}{2}}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{2x^{-\frac{1}{2}}}{4y^{-\frac{1}{2}} + 9y^{1/2}}$

15/ $\frac{d}{dx}(x + \ln(y)) = \frac{d}{dx}(x^2y^3)$

$\Rightarrow 1 + \frac{1}{y}\frac{dy}{dx} = 2xy^3 + x^2\,3y^2\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1-2xy^3}{3x^2y^2 - \frac{1}{y}}$

16/ $\frac{d}{dx}(y\ln(x) + 2) = \frac{d}{dx}(x^{3/2}y^{5/2})$

$\Rightarrow \frac{dy}{dx}\ln(x) + y\cdot\frac{1}{x} = \frac{3}{2}x^{1/2}y^{5/2} + x^{3/2}\cdot\frac{5}{2}y^{3/2}\cdot\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{3}{2}x^{1/2}y^{5/2} - y\cdot\frac{1}{x}}{\ln(x) - \frac{5}{2}x^{3/2}y^{3/2}}$

19/ $\frac{d}{dx}(x^2y^2) = \frac{d}{dx}(1)$

$\Rightarrow 2xy^2 + x^2 \cdot 2y \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2xy^2}{2x^2y} = \frac{-y}{x}$

$\frac{dy}{dx} = \frac{-1}{-1} = 1$ when $x = -1, y = 1$

Tangent line : $y - 1 = 1 \cdot (x - (-1))$

23/ $\frac{d}{dx}\left(e^{(x^2+y^2)}\right) = \frac{d}{dx}\left(xe^{5y} - y^2 e^{\frac{5x}{2}}\right)$

$\Rightarrow e^{(x^2+y^2)} \cdot \left(2x + 2y\frac{dy}{dx}\right) = 1 \cdot e^{5y} + x \cdot 5e^{5y} \cdot \frac{dy}{dx}$
$- 2y\frac{dy}{dx} e^{5x/2} - y^2 \frac{5}{2} e^{\frac{5x}{2}}$

$\Rightarrow \frac{dy}{dx} = \frac{e^{5y} - y^2 \cdot \frac{5}{2} \cdot e^{\frac{5x}{2}} - 2x e^{(x^2+y^2)}}{2y e^{(x^2+y^2)} - x \cdot 5 \cdot e^{5y} + 2y e^{5x/2}}$

$\Rightarrow \frac{dy}{dx} = 11/12$ when $x = 2, y = 1$

$\Rightarrow$ Tangent line : $y - 1 = \frac{11}{12}(x - 2)$

27/ $\frac{d}{dx}(y^3 + xy - y) = \frac{d}{dx}(8x^4)$

$\Rightarrow 3y^2\frac{dy}{dx} + 1 \cdot y + x\frac{dy}{dx} - \frac{dy}{dx} = 32x^3$

$\Rightarrow \frac{dy}{dx} = \frac{32x^3 - y}{3y^2 + x - 1}$

$x = 1 \Rightarrow y^3 + 1 \cdot y - y = 8 \cdot 1^4 \Rightarrow y^3 = 8 \Rightarrow y = 2$

$\frac{dy}{dx} = \frac{5}{2}$ when $x = 1$, $y = 2$

$\Rightarrow$ Tangent line : $(y-2) = \frac{5}{2}(x-1)$

38/ $\frac{d}{dx}(y^2) = \frac{d}{dx}(x^3 + ax + b)$

$\Rightarrow 2y\frac{dy}{dx} = 3x^2 + a \quad \Rightarrow \frac{dy}{dx} = \frac{3x^2+a}{2y}$

45/ a) $\frac{d}{dp}(\ln(q)) = \frac{d}{dp}(D - 0.44\ln(p))$

$\Rightarrow \frac{1}{q}\frac{dq}{dp} = \frac{-0.44}{p} \quad \Rightarrow \frac{dq}{dp} = \frac{-0.44q}{p}$

$\Rightarrow E = \frac{-p}{q} \cdot \frac{dq}{dp} = 0.44$

b) $\ln(q) = D - 0.44\ln(p) \Rightarrow q = e^{D - 0.44\ln(p)}$

$= e^D \cdot (e^{\ln(p)})^{-0.44}$

$= e^D \cdot p^{-0.44}$

$\Rightarrow \frac{dq}{dp} = e^D \cdot -0.44\, p^{-1.44}$

$\Rightarrow E = \frac{-p}{q}\frac{dq}{dp} = \frac{-p}{e^D p^{-0.44}} \cdot e^D \cdot (-0.44)\, p^{-1.44} = 0.44$

7.1/

5/ $\int 6\, dk = 6k + C$

7/ $\int (2z+3)\, dz = z^2 + 3z + C$

14/ $\int (t^{\frac{1}{4}} + \pi^{1/4})\, dt = \frac{1}{\frac{1}{4}+1} t^{\frac{1}{4}+1} + \pi^{1/4} t + C$

23/ $\int \pi^3 \cdot y^{-3} - \sqrt{\pi}\, y^{-\frac{1}{2}}\, dy = \pi^3 \frac{1}{-3+1} y^{-3+1} - \sqrt{\pi} \frac{1}{-\frac{1}{2}+1} y^{-\frac{1}{2}+1} + C$

36/ $\int v^2 - e^{3v}\, dv = \frac{1}{3} v^3 - \frac{1}{3} e^{3v} + C$

42/ $3^{(2x)} = (3^2)^x = 9^x$

$\Rightarrow \int 3^{(2x)}\, dx = \int 9^x\, dx = \frac{1}{\ln(9)} \cdot 9^x + C$

43/ $\int x^{2/3}\, dx = \frac{1}{2/3+1} x^{2/3+1} = \frac{3}{5} x^{5/3} + C$

$\frac{3}{5} 1^{5/3} + C = 3/5 \Rightarrow C = 0 \Rightarrow F(x) = \frac{3}{5} x^{5/3}$

49/ $\int x^{2/3} + 2\, dx = \frac{3}{5} x^{5/3} + 2x + C$

$\frac{3}{5} 8^{5/3} + 2 \cdot 8 + C = 58 \Rightarrow C = \frac{114}{5}$

$\Rightarrow C(x) = \frac{3}{5} x^{5/3} + 2x + \frac{114}{5}$

55/ $\int 500 - 0.15\, x^{\frac{1}{2}}\, dx = 500x - 0.15 \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + C$

$= 500x - (0.1)\, x^{3/2} + C$

$R(0) = 0 \Rightarrow C = 0 \Rightarrow R(x) = 500x - (0.1)\, x^{3/2}$

$R = xp = 500x - (0.1)\, x^{3/2} \Rightarrow p = 500 - 0.1\, x^{1/2}$.

58/ $\int x^{\frac{1}{2}} + \frac{1}{2}\, dx = \frac{2}{3} x^{3/2} + \frac{1}{2} x + C$

$P(0) = -1 \Rightarrow C = -1$

$\Rightarrow P(x) = \frac{2}{3} x^{3/2} + \frac{1}{2} x - 1$