MATH 16A MIDTERM 2 (PRACTICE 3)
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.
CALCULATORS ARE NOT PERMITTED
YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK
SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE
REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT
THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ______________________________

GSI's name: ______________________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) Calculate the derivatives of the following functions: (You do not need to use the limit definition and you do not need to simplify your answers)

(a)

$$x \log_2(x+1)$$

Solution:

$$\frac{d}{dx}\left(x \log_2(x+1)\right) = \frac{d}{dx}(x) \log_2(x+1) + x \frac{d}{dx}\left(\log_2(x+1)\right)$$

$$= 1 \cdot \log_2(x+1) + x \cdot \frac{1}{\ln(2)(x+1)}$$

(b)

$$\frac{3}{\sqrt{1-3^x}}$$

Solution:

$$y = 3u^{-\frac{1}{2}}, \quad u = 1 - 3^x \Rightarrow$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{-3}{2} u^{-3/2} \cdot -\ln(3) 3^x$$

$$= \frac{-3}{2}(1-3^x)^{-\frac{3}{2}} \cdot (-\ln(3) 3^x)$$

2. (25 points) A company is selling a product. The demand equation for the product is

$$p = 200e^{-0.1q}$$

where p is the price per unit and q is the number of units sold.

(a) Determine the elasticity $E(p)$.

Solution:

$$p = 200e^{-0.1q} \Rightarrow \frac{p}{200} = e^{-0.1q}$$

$$\Rightarrow q = \frac{\ln\left(\frac{p}{200}\right)}{-0.1} = -10\left(\ln(p) - \ln(200)\right)$$

$$= 10\ln(200) - 10\ln(p)$$

$$\Rightarrow \frac{dq}{dp} = \frac{-10}{p}$$

$$\Rightarrow E(p) = \frac{-p}{q} \cdot \frac{dq}{dp} = \frac{-p}{q} \cdot \frac{-10}{p} = \frac{10}{q}$$

$$= \frac{10}{10(\ln(200) - \ln(p))} = \frac{1}{\ln(200) - \ln(p)}$$

(b) If they are selling 5 units, should the company increase of decrease the price to improve revenue? Justify your answer.

Solution:

greater than 1

$E(q) = \frac{10}{q} \Rightarrow$ Elasticity is 2 if they are selling 5 units.

This means at the current price demand is elastic, hence they should decrease the price to increase revenue.

3. (25 points) Find and classify the relative extrema of the following function:

$$f(x) = 5^{(-x^2)}$$

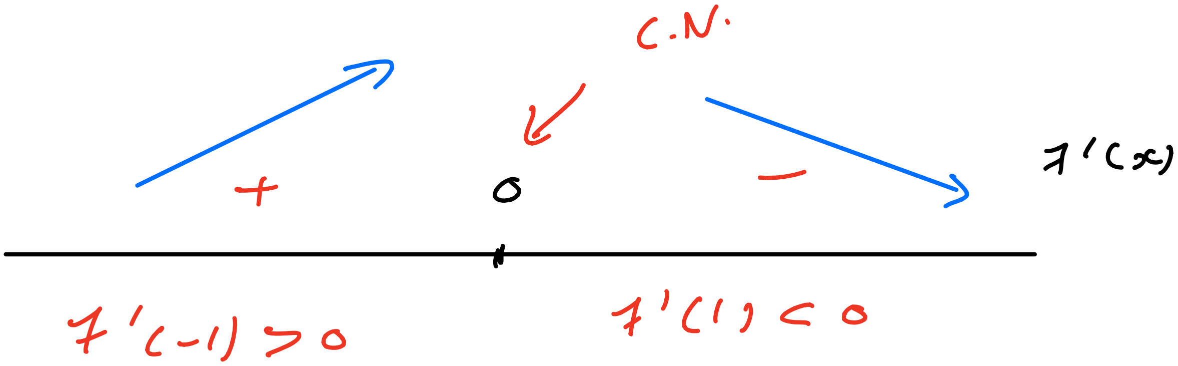
Be sure to carefully justify your answer.

Solution:

$$f'(x) = \ln(5) \cdot 5^{(-x^2)} \cdot (-2x)$$

A/ $f'(x) = 0 \Rightarrow -2x = 0 \Rightarrow x = 0$

B/ f' continuous everywhere



$\Rightarrow$ There is only one relative extrema, namely a relative max at $x = 0$.

4. Find the minimum possible value of the sum of the squares of two non-negative numbers subject to the condition that their sum is 10.

Solution:

Objective : Minimize sum of squares

$x, y \geq 0$

Objective : x^2+y^2

Constraint : $x+y = 10 \Rightarrow y = 10-x$

$\Rightarrow \quad x^2+y^2 = x^2+(10-x)^2 = f(x)$

Domain : $x \geq 0$, $x \leq 10$ $= [0,10]$

$f'(x) = 2x + 2(10-x)\cdot(-1) = 2x - 20 + 2x = 4x-20$

A/ $f'(x) = 0 \Rightarrow x = 5$

B/ f' continuous everywhere on $[0,10]$

$\Rightarrow$ 0, 5, 10 critical numbers on $[0,10]$

$f(0) = 100$
$f(5) = 50 \quad \Rightarrow$ 50 is absolute min on $[0,10]$
$f(10) = 100$

$\Rightarrow$ Minimum value of x^2+y^2 subject to $x,y \geq 0$ and $x+y=10$ is 50.

5. Sketch the following curve. If they exist, be sure to indicate asymptotes, local maxima and minima and concavity. Show your working on this page and draw the graph on the next page.

$$y = x^4 - 4x^3 \quad = f(x)$$

Solution:

Domain : $(-\infty, \infty)$

$f(0) = 0 \Rightarrow (0,0) = y$-intercept

$f(x) = 0 \Rightarrow x^3(x-4) = 0 \Rightarrow x = 0, 4 \Rightarrow (0,0), (4,0)$
x-intercepts

$\lim_{x \to \pm\infty} f(x) = \infty \Rightarrow$ No horizontal asymptotes

No vertical asymptotes

Not odd/even.

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

A/ $f'(x) = 0 \Rightarrow x = 0$ or 3

B/ f' continuous everywhere

C.N. C.N.

$-$ 0 $-$ 3 $+$ $f'(x)$

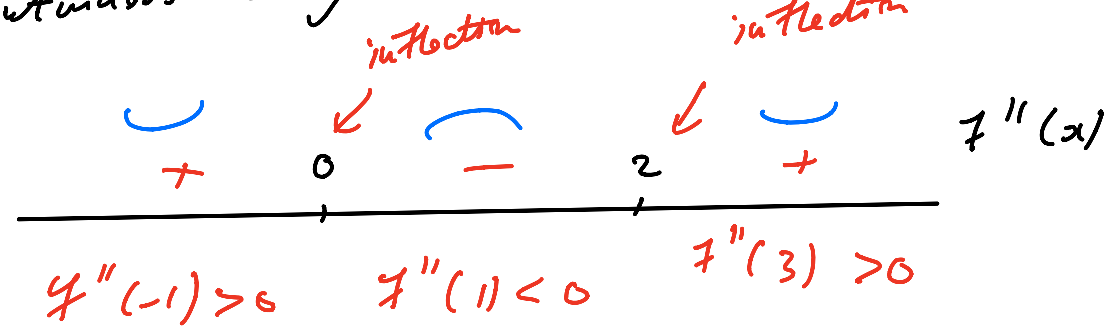
$f'(-1) < 0$ $f'(1) < 0$ $f'(4) > 0$

Solution (continued) :

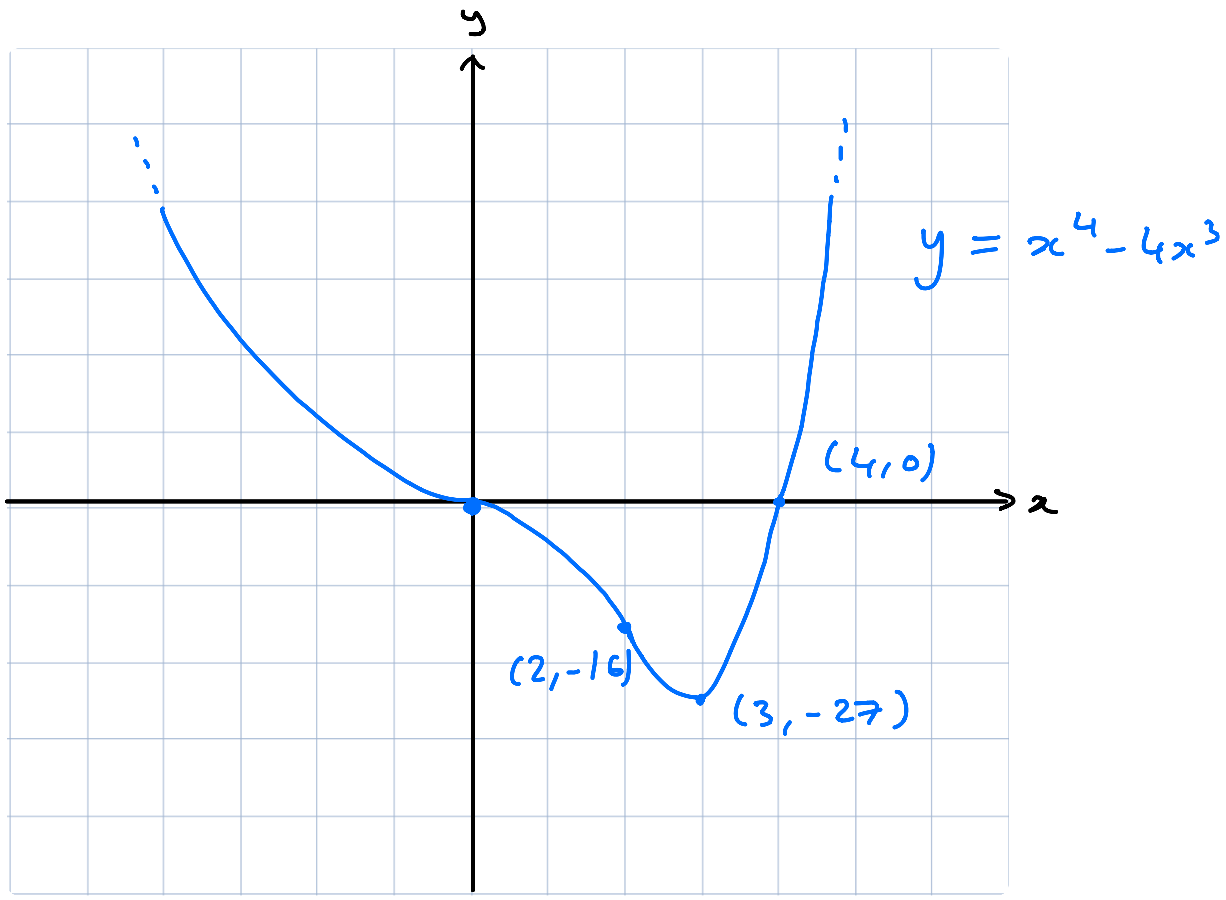
$f''(x) = 12x^2 - 24x = 12x(x-2)$

A/ $f''(x) = 0 \Rightarrow x = 0$ or 2

B/ f'' continuous everywhere



$f(0) = 0$, $f(2) = -16$, $f(3) = -27$



END OF EXAM