MATH 16A MIDTERM 2 (PRACTICE 2)

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ______________________________

GSI's name: ______________________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) Calculate the derivatives of the following functions: (You do not need to use the limit definition and you do not need to simplify your answers)

(a)

$$\ln|x^2+3|$$

Solution:

$$\frac{d}{dx} \ln|x^2+3| = \frac{2x}{x^2+3}$$

(b)

$$\frac{e^{(x^3-x)}}{1-x^3}$$

Solution:

$$\frac{d}{dx}\left(\frac{e^{(x^3-x)}}{1-x^3}\right) = \frac{\frac{d}{dx}\left(e^{(x^3-x)}\right)(1-x^3) - e^{(x^3-x)}\frac{d}{dx}(1-x^3)}{(1-x^3)^2}$$

$$= \frac{e^{(x^3-x)}\cdot(3x^2-1)\cdot(1-x^3) - e^{(x^3-x)}\cdot(-3x^2)}{(1-x^3)^2}$$

2. (25 points) A company is making and selling a product. The demand equation for a product is

$$q = 60 - 3p,$$

where p is the price per unit and q is the number of units sold. The cost of making q units is $10 + q^3$.

(a) Determine the marginal profit function.

Solution:

$q = 60 - 3p \Rightarrow p = 20 - \frac{q}{3}$

$\Rightarrow R(q) = pq = \left(20 - \frac{q}{3}\right) q = 20q - \frac{q^2}{3}$

$\Rightarrow P(q) = R(q) - C(q) = 20q - \frac{q^2}{3} - 10 - q^3$

$\Rightarrow \frac{dP}{dq} = 20 - \frac{2}{3}q - 3q^2 =$ Marginal profit function

(b) If the company is making and selling 2 units, estimate, using the marginal profit, how much extra the company will profit if they make and sell 3 units.

Solution:

$P'(q) \approx P(q+1) - P(q)$

$\Rightarrow P'(2) \approx P(3) - P(2) =$ extra profit

$P'(2) = 20 - \frac{4}{3} - 12 = 6\frac{2}{3}$

$\Rightarrow$ The will roughly be an extra profit of $6\frac{2}{3}$.

3. Determine on which intervals the following function is concave up or concave down.

$$f(x) = x^2 + \frac{1}{x} + 2$$

Be sure to carefully justify your answer. Are there any inflection points?

Solution:

$$f(x) = x^2 + \frac{1}{x} + 2$$

$$\Rightarrow f'(x) = 2x - \frac{1}{x^2}$$

$$\Rightarrow f''(x) = 2 + \frac{2}{x^3}$$

A/ $f''(x) = 0 \Rightarrow 2 + \frac{2}{x^3} = 0 \Rightarrow x^3 = -1 \Rightarrow x = -1$

B/ f'' undefined $\Rightarrow x = 0$

$f'(-1)$ exists

$f'(0)$ DNE

$+$ $\quad -1 \quad$ $-$ $\quad 0 \quad$ $+$ $\quad f''(x)$

$f''(-2) > 0$ $\quad f''(\frac{-1}{2}) < 0$ $\quad f''(1) > 0$

$\Rightarrow f$ concave up on $(-\infty, -1)$ and $(0, \infty)$

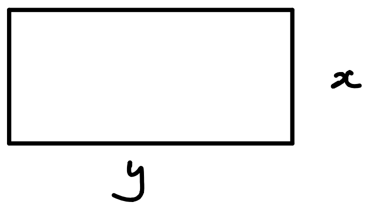
f concave down on $(-1, 0)$

There is an inflection point at $(-1, 2)$.

4. A fence is to be built which encloses a rectangular area of 20,000 ft^2. Fencing material costs 2.50 per foot for the north and south sides. Fencing material costs 3.20 per foot for the east and west sides. Find the cost of the least expensive fence.

Solution:

Objective : Minimize Cost.



Objective : $\text{Cost} = 2.5y + 2.5y + 3.2x + 3.2x$

$= 5y + (6.4)x$

Constraint : $xy = 20000 \Rightarrow y = \frac{20000}{x}$

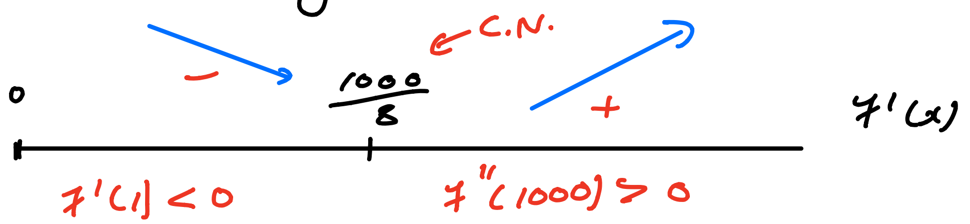
$\Rightarrow 5y + 6.4x = \frac{100000}{x} + 6.4x = f(x)$

Domain : $(0, \infty)$

$f'(x) = 6.4 - \frac{100000}{x^2}$

A/ $f'(x) = 0 \Rightarrow x^2 = \frac{100000}{6.4} = \frac{1000000}{64} \Rightarrow x = \pm \frac{1000}{8}$

B/ f' continuous everywhere on $(0, \infty)$



$\Rightarrow f\left(\frac{1000}{8}\right)$ absolute min

$\Rightarrow$ Minimum cost is $\frac{100000}{\left(\frac{1000}{8}\right)} + (6.4) \cdot \frac{1000}{8} = 800 + 800 = 1600$

5. Find and classify the relative extrema of the following function:

$$f(x) = \frac{e^{2x}}{x}$$

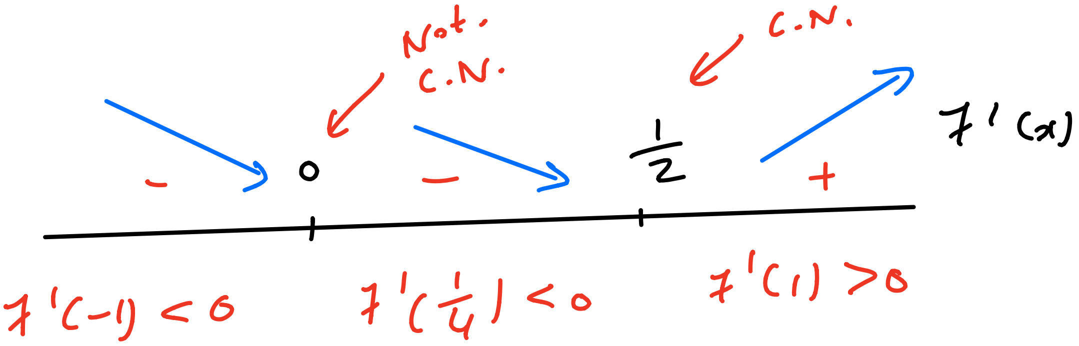
Are the results you get absolute extrema? Be sure to justify your answer.

Solution:

$$f'(x) = \frac{2e^{2x} \cdot x - e^{2x} \cdot 1}{x^2} = \underbrace{e^{2x}}_{>0} \left(\frac{2x-1}{x^2} \right)$$

A/ $f'(x) = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

B/ f' undefined $\Rightarrow x = 0$



$\Rightarrow$ The only relative extrema is at $x = \frac{1}{2}$ where there is a relative min.

The only potential absolute extrema would be an absolute min at $\frac{1}{2}$. Note that $f(\frac{1}{2}) = 2e > 0$.
However $f(-1) = -e^{-2} < 0 < f(\frac{1}{2}) \Rightarrow f(\frac{1}{2})$ <u>not</u> an absolute min.

Solution (continued) :

END OF EXAM