MATH 16A MIDTERM 2 (PRACTICE 1)

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ______________________________

GSI's name: ______________________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) Calculate the derivatives of the following functions: (You do not need to use the limit definition and you do not need to simplify your answers)

(a)

$$2^{(x^3+x+1)}$$

Solution:

$$\frac{d}{dx}\left(2^{(x^3+x+1)}\right) = \ln(2) \cdot 2^{(x^3+x+1)} \cdot \frac{d}{dx}(x^3+x+1)$$

$$= \ln(2)\, 2^{(x^3+x+1)} \cdot (3x^2+1)$$

(b)

$$\ln\left(\frac{3^x(x+1)}{x-3}\right)$$

Solution:

$$\ln\left(\frac{3^x(x+1)}{x-3}\right) = \ln(3^x) + \ln(x+1) - \ln(x-3)$$

$$= \ln(3) \cdot x + \ln(x+1) - \ln(x-3)$$

$$\Rightarrow \quad \frac{d}{dx}\left(\ln\left(\frac{3^x(x+1)}{x-3}\right)\right) = \ln(3) + \frac{1}{x+1} - \frac{1}{x-3}$$

2. (25 points) A company is selling a product. The demand equation for a product is

$$q = 100 - 2p^2,$$

where p is the price per unit and q is the number of units sold.

(a) Determine the marginal revenue when the number of units sold is 50.
Solution:

$$q = 100 - 2p^2 \Rightarrow \quad p = \sqrt{50 - \frac{q}{2}}$$

$$\Rightarrow \quad R(q) = pq = q\sqrt{50 - \frac{q}{2}}$$

$$\Rightarrow \quad R'(q) = \frac{d}{dq}(q) \cdot \sqrt{50 - \frac{q}{2}} + q \cdot \frac{d}{dq}\left(\left(50 - \frac{q}{2}\right)^{\frac{1}{2}}\right)$$

$$= \sqrt{50 - \frac{q}{2}} + q \cdot \frac{1}{2}\left(50 - \frac{q}{2}\right)^{-\frac{1}{2}} \cdot \left(\frac{-1}{2}\right)$$

$$\Rightarrow \quad R'(50) = \sqrt{50 - \frac{50}{2}} + 50 \cdot \frac{1}{2}\left(50 - \frac{50}{2}\right)^{-\frac{1}{2}} \cdot \frac{-1}{2}$$

$$= 5 + 50 \cdot \frac{1}{2} \cdot \frac{1}{5} \cdot \frac{-1}{2} = 5 - \frac{50}{20} = \frac{5}{2}$$

(b) Should the company aim to sell more or less units to increase revenue? Be sure to justify your answer
Solution:

$R'(50) > 0 \Rightarrow$ Revenue is increasing when $q = 50$

$\Rightarrow$ Company should aim to sell more than 50 to increase revenue.

3. Find, and classify, the relative extrema of the following function:

$$f(x) = x^3 - 6x^2 + 9x + 1$$

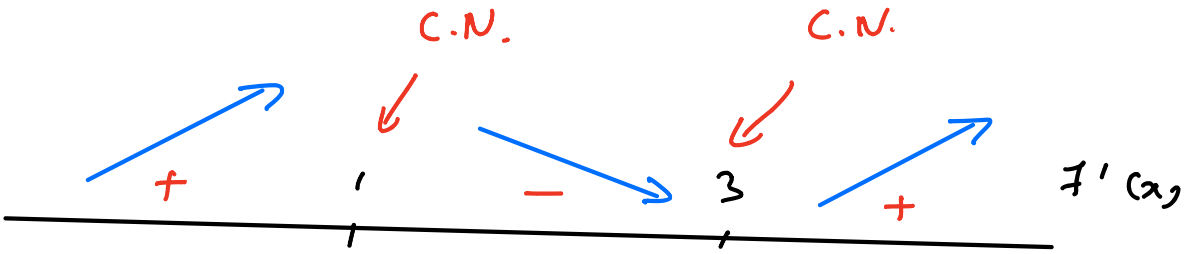
Be sure to carefully justify your answer.

Solution:

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$

A/ $f'(x) = 0 \Rightarrow x = 1$ or 3

B/ f' continuous everywhere



$f'(0) = 3 \cdot -1 \cdot -3 > 0$ $f'(2) = 3 \cdot 2 \cdot -1 < 0$ $f'(4) = 3 \cdot 1 \cdot 3 > 0$

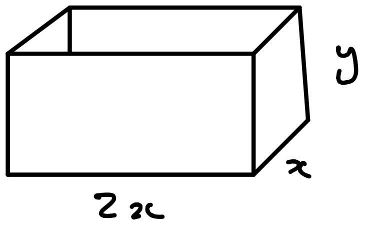
$\Rightarrow$ There is a relative maximum at $x = 1$

There is a relative minimum at $x = 3$

4. A company wishes to make a box with volume 36 ft^3 that is open on top and is twice as long as it is wide. Find the dimensions of the box which minimize the surface area. Be sure to justify your answer.

Solution:

Objective : Minimize Surface Area



Surface area : $2x^2 + 2xy + 2xy + xy + xy = 2x^2 + 6xy$

Constraint : Volume $= 36 \Rightarrow 2x^2y = 36$

$2x^2y = 36 \Rightarrow y = \frac{18}{x^2} \Rightarrow 2x^2 + 6xy = 2x^2 + 6x \cdot \frac{18}{x^2}$

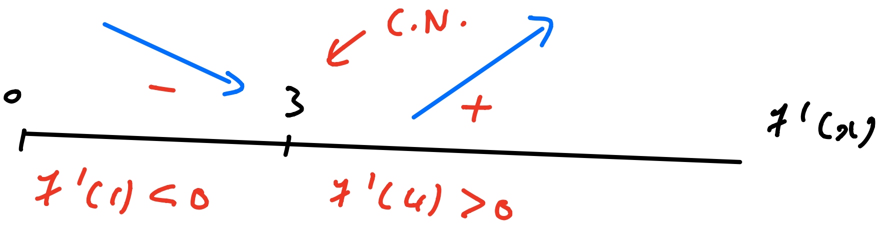
$= 2x^2 + \frac{108}{x} = f(x)$

Domain : $(0, \infty)$

$f'(x) = 4x - \frac{108}{x^2}$

A/ $f'(x) = 0 \Rightarrow x^3 = \frac{108}{4} = 27 \Rightarrow x = 3$

B/ f' continuous on $(0, \infty)$



$\Rightarrow$ Absolute min at $x = 3$ $(y = 2)$

$\Rightarrow$ $6 \times 3 \times 2$ box minimizes surface area.

5. Determine the intervals on which the following function is increasing or decreasing. Determine the intervals on which the function is concave up and concave down.

$$f(x) = \frac{x^2}{x-1}$$

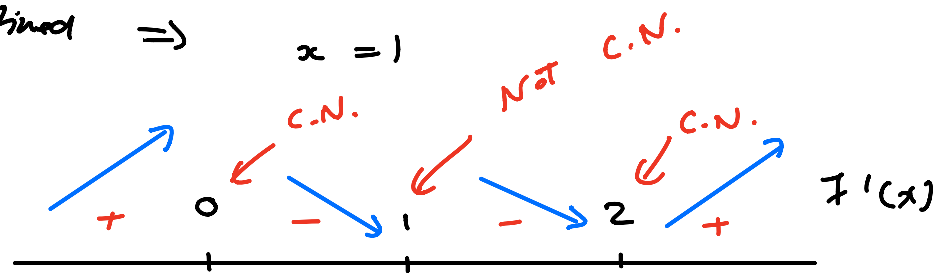
Does the graph have any inflection points?

Solution:

$$f'(x) = \frac{2x(x-1) - x^2 \cdot 1}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$$

A/ $f'(x) = 0 \Rightarrow x = 0$ or 2

B/ f' undefined $\Rightarrow x = 1$



$f'(-1) > 0$ $f'(\frac{1}{2}) < 0$ $f'(\frac{3}{2}) < 0$ $f'(3) > 0$

$\Rightarrow$ f increasing on $(-\infty, 0)$ and $(2, \infty)$

f decreasing on $(0, 1)$ and $(1, 2)$

$$f''(x) = \frac{(2x-2)(x-1)^2 - (x^2-2x) \cdot 2(x-1)}{(x-1)^4}$$

$$= \frac{(2x-2)(x-1) - 2(x^2-2x)}{(x-1)^3}$$

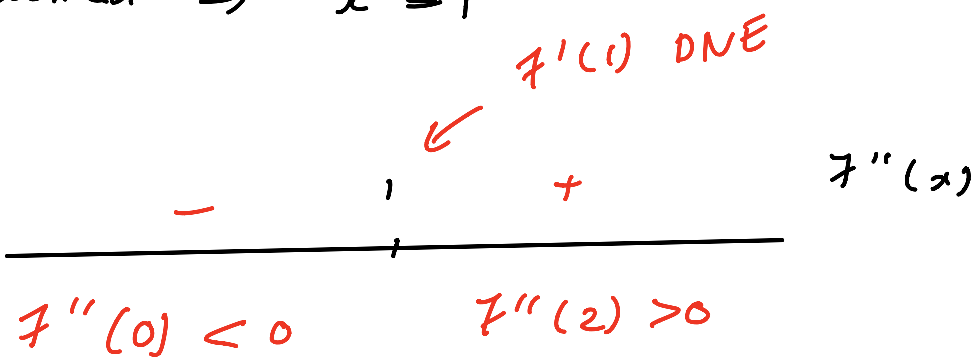
Solution (continued) :

$$\Rightarrow f''(x) = \frac{2x^2 - 4x + 2 - 2x^2 + 4x}{(x-1)^3}$$

$$= \frac{2}{(x-1)^3}$$

A/ $f''(x) = 0 \Rightarrow \dfrac{2}{(x-1)^3} = 0$ (No solutions)

B/ f'' undefined $\Rightarrow x = 1$



$\Rightarrow$ f concave down on $(-\infty, 1)$

f concave up on $(1, \infty)$

There are no inflection points.

END OF EXAM