MATH 16A MIDTERM 2 (002) 1.10PM-2PM
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.
CALCULATORS ARE NOT PERMITTED
YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK
SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE
REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT
THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ______________________________

GSI's name: ______________________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) Calculate the derivatives of the following functions: (You do not need to use the limit definition and you do not need to simplify your answers)

(a)

$$\frac{2^{(x^3)}}{x}$$

Solution:

$$\frac{d}{dx}\left(\frac{2^{x^3}}{x}\right) = \frac{\frac{d}{dx}\left(2^{x^3}\right)x - \frac{d}{dx}(x)\,2^{x^3}}{x^2}$$

$$= \frac{\ln(2)\,2^{x^3}\cdot 3x^2\cdot x - 1\cdot 2^{x^3}}{x^2}$$

(b)

$$\ln\left(\frac{\ln(x)}{x}\right)$$

Solution:

$$\ln\left(\frac{\ln(x)}{x}\right) = \ln(\ln(x)) - \ln(x)$$

$$y = \ln(\ln(x)) \Rightarrow y = \ln(u), \quad u = \ln(x)$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{u}\cdot\frac{1}{x} = \frac{1}{\ln(x)}\cdot\frac{1}{x}$$

$$\Rightarrow \frac{d}{dx}\ln\left(\frac{\ln(x)}{x}\right) = \frac{1}{x\ln(x)} - \frac{1}{x}$$

2. (25 points) A company is selling a product. The demand equation for the product is

$$q = 25 - p^2$$

where p is the price per unit and q is the number of units sold. Determine the marginal revenue when $p = \cancel{3}$ 2. Should they try and sell more or less units to increase revenue?

Solution:

$$q = 25 - p^2 \Rightarrow p = \sqrt{25 - q}$$

Revenue as a function in q

$$\Rightarrow R(q) = pq = q\sqrt{25-q}$$

$$\Rightarrow R'(q) = \frac{d}{dq}(q)\sqrt{25-q} + q \cdot \frac{d}{dq}\left(\sqrt{25-q}\right)$$

$$= \sqrt{25-q} + q \cdot \frac{1}{2} \cdot (25-q)^{-\frac{1}{2}} \cdot (-1)$$

$$= \sqrt{25-q} - \frac{q}{2\sqrt{25-q}}$$

$$= \frac{50 - 2q - q}{2\sqrt{25-q}} = \frac{50-3q}{2\sqrt{25-q}}$$

$$p = 2 \Rightarrow q = 25 - 2^2 = 21$$

$$R'(21) = \frac{50 - 3\cdot 21}{2\sqrt{25-21}} = \frac{-13}{4}$$

$R'(21) < 0 \Rightarrow$ They should try and sell fewer units to increase revenue

3. (25 points) Determine on what intervals the following function is concave up or concave down:

$$f(x) = 4x^{1/3} - x^{4/3}$$

Are there any inflection points? Be sure to carefully justify your answer.

Solution:

$$f'(x) = \frac{4}{3}x^{-2/3} - \frac{4}{3}x^{1/3}$$

$$\frac{1}{9}\frac{1}{x^{5/3}}(-8-4x)$$

$$\|$$

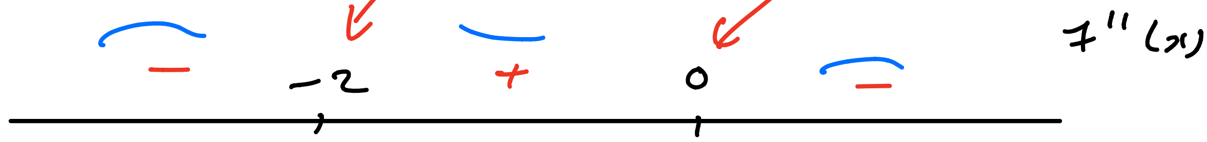
$$f''(x) = \frac{-8}{9}x^{-5/3} - \frac{4}{9}x^{-2/3} = \frac{-8}{9}\frac{1}{x^{5/3}} - \frac{4}{9}\frac{1}{x^{2/3}}$$

A/ $f''(x) = 0 \Rightarrow \frac{-8}{9}\frac{1}{x^{5/3}} = \frac{4}{9}\frac{1}{x^{2/3}} \Rightarrow x = -2$

B/ f'' undefined $\Rightarrow x = 0$

$f'(-2)$ exists

$f'(0)$ DNE



$f'(-8) < 0$ $f''(1) < 0$

$f''(-1) > 0$

$\Rightarrow$ f concave up on $(-2, 0)$

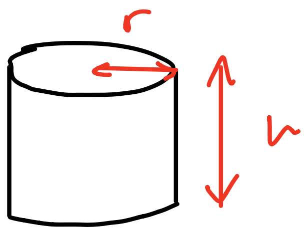
f concave down on $(-\infty, -2)$ and $(0, \infty)$

$\left(-2,\ 4(-2)^{1/3} - (-2)^{4/3}\right)$

4. A company plans to package its product in a cylinder that is open at one end. The cylinder must have volume 27π cm^3. What radius should the cylinder be to minimize the surface area?

Solution:

Objective : Minimize surface area



Objective : $\pi r^2 + 2\pi r h$

Constraint : Volume $= 27\pi$

$\Rightarrow \pi r^2 h = 27\pi$

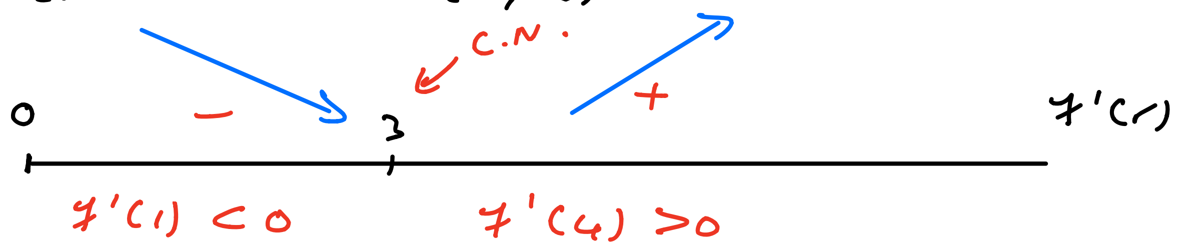
$\Rightarrow h = \frac{27}{r^2} \Rightarrow \pi r^2 + 2\pi r h = \pi r^2 + \frac{54\pi}{r} = f(r)$

Domain : $r \neq 0,\ r \geq 0 \quad = (0, \infty)$

$f'(r) = 2\pi r - \frac{54\pi}{r^2}$

A/ $f'(r) = 0 \Rightarrow r^3 = 27 \Rightarrow r = 3$

B/ f' continuous on $(0, \infty)$



$\Rightarrow f(3)$ absolute min on $(0, \infty)$

$\Rightarrow$ To minimize the surface area the radius should by 3 cm.

5. Sketch the following curve. If they exist, be sure to indicate relative maxima and minima and inflection points. Show your working on this page and draw the graph on the next page.

$$y = -x + \frac{1}{1-x}$$

Solution:

$$f(x) = -x + \frac{1}{1-x}$$

Domain : $x \neq 1$

y-intercept : $(0, 1)$

$\lim_{x \to \infty} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = +\infty$ $\Rightarrow$ no horizontal asymptotes

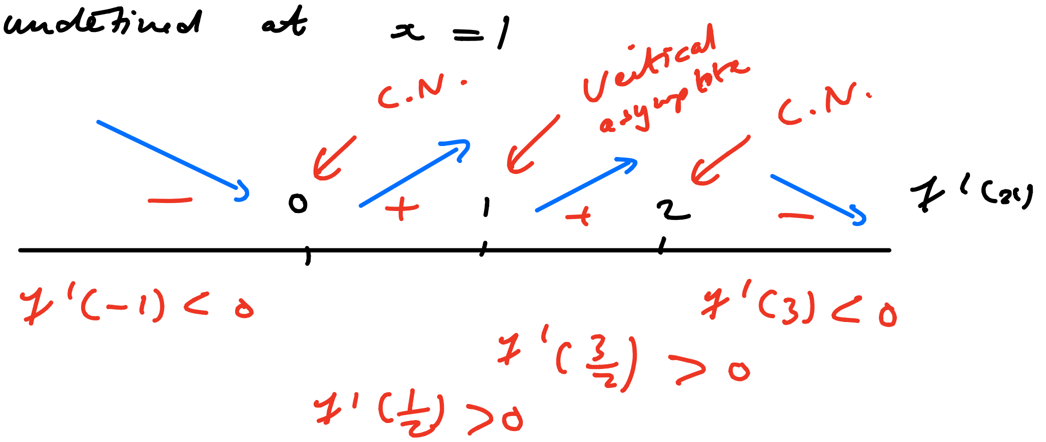
Vertical asymptote at $x = 1$

Neither odd nor even

$$f'(x) = -1 + \frac{1}{(1-x)^2}$$

A/ $f'(x) = 0 \Rightarrow (1-x)^2 = 1 \Rightarrow x = 0$ or 2

B/ f' undefined at $x = 1$



$$f''(x) = \frac{2}{(1-x)^3}$$

Solution (continued) :

A/ $f''(x) = 0 \Rightarrow \dfrac{2}{(1-x)^3} = 0$ (No solutions)

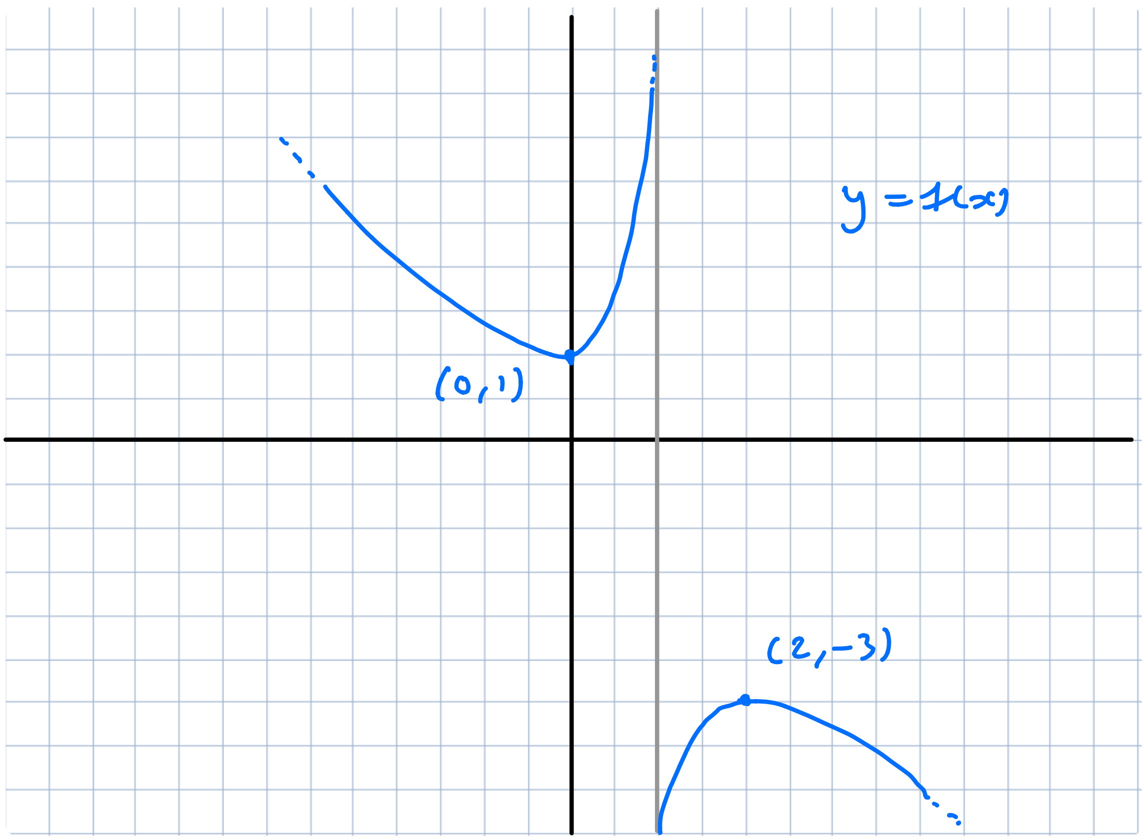
B/ f'' undefined $\Rightarrow x = 1$

Not inflection (vertical asymptote)

$+$ $\quad 1 \quad$ $-$ $\quad f''(x)$

$f''(0) > 0$ $\quad$ $f''(2) < 0$

$f(0) = 1$, $f(2) = -3$



END OF EXAM