MATH 16A FINAL EXAM (PRACTICE 2)
PROFESSOR PAULIN

<table>
<tr><td>DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.</td></tr>
<tr><td>CALCULATORS ARE NOT PERMITTED</td></tr>
<tr><td>YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK</td></tr>
<tr><td>REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT</td></tr>
<tr><td>THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM</td></tr>
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Name and section: _____

GSI's name: _____

> **This exam consists of 10 questions. Answer the questions in the spaces provided.**

1. Calculate the following derivatives (you do not need to use limits):

   (a)
   $$\frac{d}{dx}(x\sqrt{x} + 3)$$

   Solution:

   (b)
   $$\frac{d}{dt}\left(\frac{t^2 + 1}{t^3 - 2}\right)$$

   Solution:

   (c)
   $$\frac{d^2}{dx^2}(3^{\sqrt{x}})$$

   Solution:

2. Calculate the following integrals:

   (a)
   $$\int (x^3 - x)dx$$

   Solution:

   (b)
   $$\int \frac{\sqrt{x} - 1}{\sqrt{x}}dx$$

   Solution:

   (c)
   $$\int_1^2 \frac{2^{(1/u)}}{u^2}du$$

   Solution:

3. Using the limit definition, calculate the derivative of $f(x) = \frac{2}{x^2} + x$.

   Solution:

4. A product is being sold. The demand equation is $q^2 + qp + 4p^2 = 10$, where p is the price per unit and q is the number of units sold.

   (a) Calculate the elasticity. Your answer should involve both p and q.

   Solution:

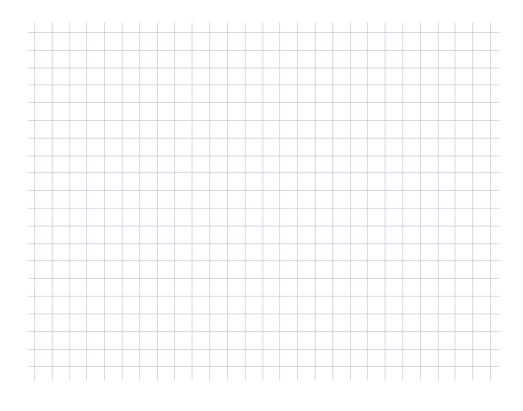   (b) If $q = 3$ should they increase or decrease the price to raise revenue?

   Solution:

5. Determine the concavity of the following function:

$$f(x) = x^2 + 8\ln|x+1|.$$

Are there any inflection points? If so, find them.

Solution:

$$f(x) = x^2 + 8\ln|x+1|.$$

6. Sketch the following curve. If they exist, be sure to indicate relative extrema and inflection points. Show your working on this page and draw the graph on the next page.

$$y = \frac{1}{x^2 + 4x + 3}$$

Solution:

7. An open box will be made by cutting a square from each corner of a 3 by 8 foot piece of cardboard and then folding up the sides. What size squares should be cut from each corner to maximize the volume?

   Solution:

8. A company incurs debt at a rate of

$$(2t + 3)\sqrt{t + 1}$$

dollars per year, where t is the amount of time (in years) since the company started. How much will the company's debts have grown between $t = 3$ and $t = 8$?

Solution:

9. Determine the area of the region enclosed by the x-axis and the curve

$$y = \begin{cases} -2 - x & \text{if } x < 0 \\ x^2 - 2 & \text{if } x \geq 0 \end{cases}$$

between -3 and 2.

Solution:

10. Let $f(x) = x^4 + x^3 + x^2 - 2x + 1$ and $g(x) = x^4 + x^2 - x + 1$. Calculate the total area of the region bounded by $y = f(x)$ and $y = g(x)$.

Solution:

END OF EXAM