MATH 110

PROFESSOR KENNETH A. RIBET

First Midterm Examination

February 20, 2014

9:40–11:00 AM, 105 Stanley Hall

Please write your NAME clearly:

Please put away all books, calculators, cell phones and other devices. You may consult a single two-sided sheet of notes. Please write carefully and clearly in *complete sentences*. Your explanations are your only representative when your work is being graded.

Unless otherwise noted, vector spaces are vector spaces over $\mathbf{F}$, where $\mathbf{F} = \mathbf{R}$ or $\mathbf{F} = \mathbf{C}$.

At the conclusion of the exam, hand your paper in to your GSI.

Problem	Your score	Possible points
1		5 points
2		6 points
3		6 points
4		6 points
5		7 points
Total:		30 points

1. Consider the basis $(1, x, x^2, x^3)$ of the $\mathbf{R}$-vector space $V = \mathcal{P}_3(\mathbf{R})$. Let $(\varphi_1, \ldots, \varphi_4)$ be the basis of V' that is dual to $(1, x, x^2, x^3)$. Let $\varphi : V \to \mathbf{R}$ be the linear functional

$$f(x) \mapsto f(6) + \int_0^1 f(x)\, dx.$$

Find numbers a, b, c, d for which $\varphi = a\varphi_1 + b\varphi_2 + c\varphi_3 + d\varphi_4$.

2. Label each of the following assertions as TRUE or FALSE. Along with your answer, provide an informal proof, counterexample or other explanation.

a. If $T : V \to W$ is a linear map and $v_1, v_2, \ldots, v_r$ are vectors of V such that $(Tv_1, \ldots, Tv_r)$ is linearly independent, then $(v_1, \ldots, v_r)$ is linearly independent.

b. If $T : V \to W$ is a linear map and $v_1, v_2, \ldots, v_r$ are vectors of V such that $(Tv_1, \ldots, Tv_r)$ spans W, then $(v_1, \ldots, v_r)$ spans V.

3. Label each of the following assertions as TRUE or FALSE. Along with your answer, provide an informal proof, counterexample or other explanation.

a. If X is a 5-dimensional subspace of a 8-dimensional vector space V, there is a 2-dimensional subspace Y of V such that $X \cap Y = \{0\}$.

b. If $(v_1, \ldots, v_m)$ and $(w_1, \ldots, w_m)$ are linearly independent lists of vectors in V, then $(v_1 + w_1, \ldots, v_m + w_m)$ is linearly independent.

4. Suppose that $p_0, p_1, \ldots, p_m$ are polynomials in $\mathcal{P}_m(\mathbf{F})$ such that $p_j(-1) = 0$ for all j. Prove that $(p_0, p_1, \ldots, p_m)$ is not linearly independent in $\mathcal{P}_m(\mathbf{F})$.

5. Let U be a subspace of V and let $T : U \to W$ be a non-zero linear map. Suppose that the function

$$S(v) := \begin{cases} T(v) & \text{if } v \in U \\ 0 & \text{if } v \notin U \end{cases}$$

is a linear map $V \to W$. Show that $U = V$.