# Math 110

**PROFESSOR KENNETH A. RIBET**

## First Midterm Examination

## February 16, 2010

2:10–3:30 PM, 10 Evans Hall

Please write your NAME clearly:

Please put away all books, calculators, and other portable electronic devices—anything with an ON/OFF switch. You may refer to a single 2-sided sheet of notes. For numerical questions, *show your work* but do not worry about simplifying answers. For proofs, write your arguments in complete sentences that explain what you are doing. Remember that your paper becomes your only representative after the exam is over.

At the conclusion of the exam, hand your paper in to your GSI.

| Problem | Your score | Possible points |
|---|---|---|
| 1 | | 5 points |
| 2 | | 12 points |
| 3 | | 6 points |
| 4 | | 7 points |
| Total: | | 30 points |

**1.** In $\mathbf{R}^3$, express $(3, 18, -11)$ as a linear combination of $(1, 2, 3)$, $(-2, 0, 3)$ and $(2, 4, 1)$.

**2.** Label each of the following statements as TRUE or FALSE. Along with your answer, provide an informal proof or an explanation. For false statements, an explicit counterexample might work best. In interpreting the statements, take $v$ to be a vector, $a$ to be a scalar, $\beta$ to be a basis of $V$, etc., etc.

**a.** If $av = v$, then either $a = 1$ or $v = 0$.

**b.** If $A$ and $B$ are real $3 \times 3$ matrices, the formula $T(M) = AM - MB$ defines a linear map $\mathsf{M}_{3\times 3}(\mathbf{R}) \to \mathsf{M}_{3\times 3}(\mathbf{R})$.

**c.** If $V$ is spanned by a set of 6 distinct vectors, all bases of $V$ have exactly 6 vectors.

**d.** If $W$ is a subspace of a finite-dimensional vector space $V$ and $w_1, \ldots, w_m$ form an ordered basis of $W$, then every basis of $V$ includes $w_1, \ldots, w_m$.

**e.** In $\mathcal{L}(F^6, F^4)$, one may find linear transformations $T$ for which the dimensions of $\mathsf{N}(T)$ are 2, 3, 4, 5 and 6.

**f.** If $m = \dim(V)$ and $n = \dim(W)$, then $[T]_\beta^\gamma$ is an $n \times m$ matrix. (Here $T$ is a linear map $V \to W$.)

**3.** Suppose that $V$ is an $F$-vector space with at least three vectors. Let $w$ be a vector in $V$. Prove that $V$ is spanned by the set $S = \{\, v \in V \mid v \neq w \,\}$.

**4.** Let $f(x)$ be a polynomial of degree $n$ with real coefficients. Prove that the $n+1$ polynomials

$$f(x), f'(x), f''(x), \ldots, f^{(n)}(x)$$

are linearly independent. Conclude that they span $\mathsf{P}_n(\mathbf{R})$.