Math 10B first midterm, February 16, 2016

NAME: ______________________ SID: ______________

1. A child rolls a fair die six times in succession. What is the probability that she rolled each of the numbers 1–6 exactly once?

2. Calculate the number of ways of rearranging the letters `CALBEARS` so that `E` appears somewhere to the left of `S`.

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

NAME: ______________________ SID: ______________

3. How many ways are there to put 12 indistinguishable burritos and 12 indistinguishable tacos into six distinguishable boxes?

NAME: ______________________ SID: ______________

4. In how many ways can three students be assigned to six different discussion sections if each discussion section can receive at most one student?

5. In how many ways can six students be assigned to three different discussion sections, if each discussion section must receive at least one student?

NAME: ______________________ SID: _______________

6. For $n \geq 0$, let $F(n)$ be the number of bit strings of length n that do not contain `00`. We have $F(0) = 1$, $F(1) = 2$, $F(2) = 3, \ldots$.

a. For $n \geq 2$, explain why the number of bit strings of length n that start with `0` and do not contain `00` is equal to $F(n-2)$.

b. For $n \geq 1$, explain why the number of bit strings of length n that start with `1` and do not contain `00` is equal to $F(n-1)$.

c. Calculate the number of bit strings of length 4 that do not contain `00`, using the insights of parts **a** and **b** (or otherwise, if you prefer).

NAME: ______________________ SID: _______________

7. Before going on vacation for a week, you ask your spacey friend to water your ailing plant. Without water, the plant has a 90 percent chance of dying. With water, it has a 20 percent chance of dying. The probability that your friend will forget to water it is 30 percent.

a. What is the probability that your plant will be dead at the end of the week?

b. If the plant is dead when you return, what is the probability that your friend forgot to water it?

Math 10B second midterm, March 31, 2016

NAME: ______________________ SID: _______________

1. Find an explicit formula for a_n $(n \geq 0)$ if

$$a_n = a_{n-1} + 6a_{n-2} \text{ (for } n \geq 2), \quad a_0 = -1, \quad a_1 = 22.$$

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

NAME: ______________________ SID: _______________

2. Find a function $y(t)$ $(t > 0)$ such that $ty' + 2y = 4t^2$ and $y(1) = 0$.

NAME: ______________________ SID: _______________

3a. A single card is to be drawn from a standard deck of 52 cards, examined and replaced. This procedure will be repeated four more times, for a total of five draws. What is the expected number of diamonds that will be drawn? [Note that there are 13 diamonds in the deck.]

b. If a 5-card poker hand is drawn at random from a standard deck of 52 playing cards, what is the expected number of diamonds in the hand?

NAME: ______________________ SID: _______________

(Scratchwork)

Math 10B second midterm, March 31, 2016

NAME: ______________________ SID: ______________

4. Write as a sum of partial fractions: $\dfrac{8x^3+2x^2+42x+8}{(x-1)(x+1)(x^2+9)}$.

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

NAME: ______________________ SID: ______________

5a. We suspect that a coin is biased so that it comes up heads 2/3 of the time. Flipping the coin 6000 times, we obtain 3929 heads and 2071 tails. To test the *null hypothesis* that the coin is biased in the way that we suspect, what χ^2 test statistic do we compute? (Your answer should be an explicit number, but you don't have to simplify it or estimate its value.)

b. After computing the test statistic, we obtain 0.0518 as the corresponding p-value. What should we conclude about the null hypothesis?

Math 10B second midterm, March 31, 2016

NAME: ______________________ SID: ______________

6. "A company makes electric motors. The probability that an electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors?"

a. Calculate this probability (but don't bother to evaluate your answer as a decimal number).

b. What estimate for this probability would be supplied to us by the Poisson distribution?

NAME: ______________________ SID: _______________

1. How many 5-card poker hands have at least two face cards? (The face cards are the jacks, queens and kings. Thus there are 12 face cards in a standard 52-card deck.)

NAME: ______________________ SID: _______________

2. In some population, the probability that a woman has hemophilia is 0.5. If a woman has the hemophilia gene, then each of her sons independently has the probability 0.5 of having the disease; if she does not have the gene, then none of her sons will have the disease. None of Alice's three sons has hemophilia. What is the probability that Alice carries the hemophilia gene?

NAME: ______________________ SID: _______________

3. Find all solutions to the differential equation $y'' - 4y' + 5y = 0$ that satisfy $y(0) = y(\pi) = 10$.

NAME: ______________________ SID: _______________

4a. Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$. Find a non-zero vector v such that $Av = v$.

b. For the v that you found in part (a), find a vector w such that $Aw = w + v$.

NAME: ______________________ SID: _______________

5. Suppose that A is an $n \times n$ real matrix and that v is an eigenvector of A with eigenvalue λ. Suppose also that w is a vector of real numbers such that $Aw = \lambda w + v$. Show that $y = e^{\lambda t}(tv + w)$ is a solution to the system of differential equations $y' = Ay$.

NAME: ______________________ SID: _______________

6. The dials of RSF combination locks display numbers from 0 to 39. Combinations are lists of three numbers for which the differences between successive numbers are even, but not divisible by 4. Thus 30–4–10 is an allowable combination because $30 - 4 = 26$ and $10 - 4 = 6$ are even numbers but not multiples of 4. Other allowable combinations are 21–15–17, 0–10–0, 1–7–33; note that the three numbers in an allowable combination are either all odd or all even. Combinations that are *not* be permitted include 31–5–33 (because $33 - 5 = 28$ is a multiple of 4) and 10–2–8 (because $10 - 2 = 8$ is a multiple of 4).

Assume that RSF patrons are assigned locks randomly and independently, with all allowable combinations equally likely. Find the probability that the first ten patrons who requested locks today were given ten different combinations.

NAME: ______________________ SID: _______________

7. Find the inverse of the matrix $\begin{pmatrix} 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 0 \end{pmatrix}$.

NAME: ______________________ SID: ______________

8. Calculate $\displaystyle\int \frac{x^3 + 6x^2 - 5x + 2}{x^3 - x^2}\,dx.$

NAME: ______________________ SID: _______________

9. The expansion of $(x+y+z)^3$ contains ten terms:

$$(x+y+z)^3 = x^3 + 3x^2y + 3xy^2 + y^3 + 3x^2z + 6xyz + 3y^2z + 3xz^2 + 3yz^2 + z^3.$$

How many terms are there in the expansion of $(x+y+z+w)^{100}$?