Math 10A midterm exam, October 26, 2017

You put away all books, calculators, cell phones and other devices. You consulted a single two-sided sheet of notes. You wrote carefully and clearly, *USING WORDS* (not just symbols). The paper you handed in was your only representative when your work is graded.

Point counts:

Problem	1	2	3	4	5	6	7	Total
Points	5	5	6	6	6	6	6	40

1. The perimeter of a regular n-gon inscribed in the circle of radius 1 is $2n\sin(\frac{\pi}{n})$. Find the limit as $n \to \infty$ of this expression. (Explain in words what you are doing—this requirement applies to each of the questions on this midterm.)

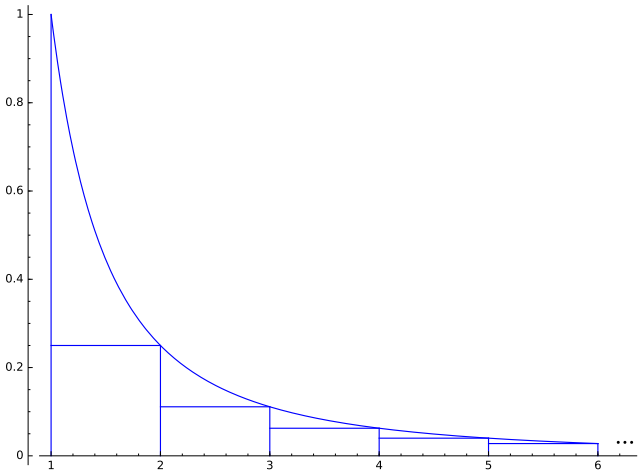
2a. Show that $\dfrac{1}{m^2+3m+2} = \dfrac{1}{m+1} - \dfrac{1}{m+2}$ for $m \geq 0$.

b. Find the sum of the infinite series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$ by considering the partial sums of the series.

3. Referring to the diagram below, explain carefully why

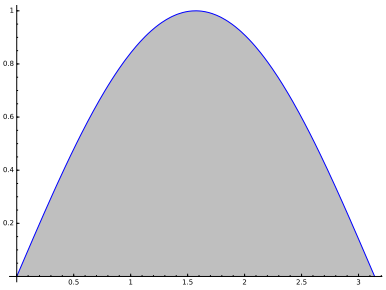
$$\frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 1 - \frac{1}{n}$$

for $n \geq 2$.



You acted with honesty, integrity, and respect for others.

4. Determine the volume of the solid obtained by revolving the area under $y = \sin x$ from $x = 0$ to $x = \pi$ about the x-axis. [Hint: it may be helpful to know that $\cos 2x = 1 - 2\sin^2 x$.]



5. Find y as a function of x, given $\dfrac{dy}{dx} = y(2x + 1)$ and $y(0) = 2$.

6. Use integration by parts, twice, to find an antiderivative of $e^x \sin x$.

7. Use the chain rule and the fundamental theorem of calculus to find

$$\frac{d}{dx}\left(\int_{x^2}^{x^3} \sin(t^2)\, dt\right).$$