Math 250A

PROFESSOR KENNETH A. RIBET

First Midterm Examination

September 30, 2004

12:40–2:00 PM

Name:

Please put away all books, calculators, electronic games, cell phones, pagers, .mp3 players, PDAs, and other electronic devices. You may refer to a single 2-sided sheet of notes. Explain your answers in full English sentences as is customary and appropriate. Your paper is your ambassador when it is graded.

Problem:	Your score:	Total points
1		6 points
2		6 points
3		6 points
4		6 points
5		6 points
Total:		30 points

1. If n is a positive integer, find an $m \geq 1$ so that the alternating group $\mathbf{A}_m$ contains a subgroup isomorphic to the symmetric group $\mathbf{S}_n$.

2. Prove that every group of order $312 = 2^3 \cdot 3 \cdot 13$ has a proper non-trivial normal subgroup.

3. Let G be a group of order 120, and let $H \subseteq G$ be a subgroup of order 24. Suppose that there is at least one left coset of H in G (other than H itself) that is also a right coset of H in G. Prove that H is a normal subgroup of G.

4. Let G be a group and let H be a subgroup of G such that the index $(G : H)$ is finite. Prove that there is a normal subgroup H_0 of G such that $H_0 \subseteq H$ and such that $(G : H_0)$ is finite. Show further that there is an $n \geq 1$ so that $g^n \in H$ for all $g \in G$.

5. Let G be a finite group, and let H be a normal subgroup of G. Let P be a p-Sylow subgroup of H, and let K be the normalizer of P in G. Establish the equality $G = HK$.