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A Gentle Introduction to (Equivariant) Morse Theory

ABSTRACT

We will begin by reviewing the Morse theory of the torus $T^2 = S^1 \times S^1$, using the height function as our perfect Morse function (see figure below). We will see that this function is also an equivariantly perfect Morse function, with respect to a $\mathbf{Z}_2 \times \mathbf{Z}_2$ action. One reason that this is true is that we may view $T^2 = S^1 \times S^1$ as the real points $\mathbf{RP}^1 \times \mathbf{RP}^1$ of the complex variety $\mathbf{CP}^1 \times \mathbf{CP}^1$. We will describe some results relating the (equivariant) topology of a symplectic manifold to the (equivariant) topology of its real points. In particular, in the case of T^2 , we will show how to answer a question arising in string theory.