George M. Bergman
102 Lewis and 60 Evans

Spring 2001, Math 53M

Second Midterm

16 March, 2001
10:10-11:00 AM

1. (66 points, 11 points apiece) Find the following. If an expression is undefined, say so.

(a) $\frac{d}{dt}\ F(t,\, 3t^2,\, 5)$, where F is a differentiable function of three variables. Express your answer in terms of the partial derivatives of F.

(b) A unit normal vector to the surface $(x+y)^3 - (y+z^2)^5 = 9$ at the point $(4,-2,1)$.

(c) $\int_1^2 \int_0^{1/x} e^{xy}\, dy\, dx$.

(d) $\int\int_D (x/y^2)\, dA$, where D is the rectangle $-1 \leq x \leq 2,\ 1 \leq y \leq 2$.

(e) $\int\int_E (x/y^2)\, dA$, where E is the rectangle $1 \leq x \leq 2,\ -1 \leq y \leq 2$.

(f) $\int_0^2 \int_{x^2}^{2x} F(x,y)\, dy\, dx$, expressed as a double integral with the order of integration reversed (where F is a continuous function).

2. (20 points) Find the maximum and minimum values of the function $(x^2+2y^2)^{1/2}$ on the disk $D = \{(x,y) \mid (x-1)^2 + y^2 \leq 9\}$, and the points at which they occur.

3. (14 points) Let R be the rectangle $\{(x,y) \mid a \leq x \leq b,\ c \leq x \leq d\}$, for real numbers $a < b$ and $c < d$. Compute the average value of the function xy over this rectangle R, and show by computation that it is equal to the value of that function at the midpoint $((a+b)/2,\, (c+d)/2)$ of R.