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Spring 2001, Math 53M
First Midterm

16 February, 2001
10:10-11:00 AM

1. (54 points, 9 points apiece) Find the following. If an expression is undefined, say so.

(a) dy/dx, where $x = 2\sin t$, $y = 3\cos t$. Express your answer as a function of t.

(b) The area of the region between the curve whose expression in polar coordinates is $r = e^{\theta}$ $(0 \leq \theta \leq \pi/2)$, the line $\theta = 0$, and the line $\theta = \pi/2$.

(c) $\lim_{(x,y)\to(0,0)} x/y$.

(d) The equation of the plane tangent to the surface $z = (x+y)^{½}$ at the point where $x = 2$, $y = 7$.

(e) $\frac{\partial^2}{\partial x \partial y}(f(x)\,g(y))$, where f and g are differentiable functions. (Express your answer in terms of f and g and their derivatives.)

(f) $\int_0^1 (\mathbf{j} \times (t^2\mathbf{i} + e^{-t^2}\mathbf{j} + (\tan t)\mathbf{k})\,dt$ (where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the standard basis vectors in $\mathbb{R}^3$).

2. (34 points) **(a)** (20 points) Let f be a continuously differentiable real-valued function on the interval $[-\pi, \pi]$. Show that the space curve given by the vector equation $\mathbf{r}(t) = \langle f(t), \sin t, \cos t\rangle$, $(-\pi \leq t \leq \pi)$ has the same arc-length as the curve $y = f(x)$ $(-\pi \leq x \leq \pi)$ in the plane. You may assume formulas for arc length given in Stewart.

(b) (14 points) Find the length of the curve $\mathbf{r}(t) = \langle \surd(\pi^2 - t^2), \sin t, \cos t\rangle$, $(-\pi \leq t \leq \pi)$. You may use the result of part (a) whether or not you have proved it; or you may use any other method that gives the correct answer.

3. (12 points) Find equations in *cylindrical* and *spherical* coordinates for the sphere described in Cartesian coordinates by the equation $x^2 + y^2 + (z-1)^2 = 1$.