Mathematics Department Colloquium

Organizer: John Strain

Thursday, 4:10–5:00pm, 60 Evans

Nov 16 **N. Christopher Phillips**, University of Oregon

Crossed products of irrational rotation algebras by finite groups

For each $\theta \in \mathbb{R}$, the rotation algebra A_θ is defined to be the universal C*-algebra generated by two unitary operators u and v satisfying $vu = e^{2\pi i\theta}uv$. If $\theta = 0$, this C*-algebra is just $C(S^1 \times S^1)$, the algebra of continuous complex valued functions on the torus. Hence, for general θ this C*-algebra is sometimes called a noncommutative torus.

There is an action of $\mathrm{SL}_2(\mathbb{Z})$ on A_θ which generalizes the action of $\mathrm{SL}_2(\mathbb{Z})$ on $S^1 \times S^1$ obtained via the identification $S^1 \times S^1 \cong \mathbb{R}^2/\mathbb{Z}^2$. In particular, the subgroups of $\mathrm{SL}_2(\mathbb{Z})$ isomorphic to $\mathbb{Z}/2\mathbb{Z}$, $\mathbb{Z}/3\mathbb{Z}$, $\mathbb{Z}/4\mathbb{Z}$, and $\mathbb{Z}/6\mathbb{Z}$ all act on A_θ. For example, the action of $\mathbb{Z}/4\mathbb{Z}$ is generated by the automorphism which sends u to v and v to u^*.

For rational θ, the K-theory of the crossed products of A_θ by these groups has been calculated. The results suggest the conjecture that, for θ irrational, the crossed products are AF algebras, that is, direct limits of finite dimensional C*-algebras. This has been known for some time for $\mathbb{Z}/2\mathbb{Z}$. We prove this conjecture for the other three groups. One might hope to be able to write down an approximating sequence of finite dimensional subalgebras. In fact, the proof uses both the Elliott classification program and the Baum-Connes Conjecture, two central areas of research in C*-algebras which previously have had little contact.

In this talk, I will describe the C*-algebras involved, and give some idea of the role that the Elliott classification program and the Baum-Connes Conjecture play in the proof. It is intended to be accessible to people from outside the area.

This is joint work with Siegfried Echterhoff, Wolfgang Lück, and Sam Walters.