Mathematics Department Colloquium

Organizer: Olga Holtz

Thursday, 4:10–5:00pm, 60 Evans

Oct. 30   **Andrew Hassell**, Australian National University
          Quantum Billiards

Abstract: By a "billiard" I mean a bounded plane domain D, with smooth (enough) boundary. Quantum billiards is the study of properties of eigenfunctions of the Laplacian on D, i.e. solutions of $\Delta u = Eu$, where u is a function on D vanishing at the boundary, Δ is the Laplacian on D and E is a real number, in the limit as $E \to \infty$. This large-E limit is the "classical limit" in which eigenfunctions exhibit behaviour related to the classical billiard system (a billiard ball moving around inside D, bouncing elastically off the boundary). I will talk about Quantum Ergodicity, which is the property that "most of" the eigenfunctions become uniformly distributed in D, asymptotically as $E \to \infty$, i.e. they are the same size, on average, in all parts of the domain D; and the stronger property of Quantum Unique Ergodicity, which is the same property with the words "most of" deleted.

In this talk I will discuss a partial solution to a conjecture made about 20 years ago, that certain quantum ergodic domains called "stadium domains" are not Quantum Unique Ergodic. I will motivate and discuss this conjecture and talk a little about the proof, which is surprisingly simple.