On solvability and attractors of the Navier-Stokes equations

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Abstract

I deal with the systems

$$v_t + v \cdot \nabla v - \mathrm{div}\sigma(\varepsilon(v)) + \nabla \rho = f, \quad \mathrm{div} v = 0, \tag{1}$$

where

$$\sigma(\varepsilon) = \frac{\partial D(\varepsilon)}{\partial \varepsilon}, \tag{2}$$

and D is a given function characterizing the (incompressible) fluid.

For

$$D(\varepsilon) = 2\nu\varepsilon \tag{3}$$

with $\nu = \mathrm{const} > 0$, (1) is the Navier-Stokes system, describing the dynamics of Newtonian fluids when gradients of the velocity field v are "not large". For the so-called generalized Newtonian fluids, ν in (3) is a function of $|\varepsilon|$.

I present the results about solvability of some initial-boundary value problems for (1), (2), the behavior of their solutions when $t \to \infty$, and the estimation of the fractal dimension of the minimal global B-attractors for these problems.