Macdonald polynomials, Hilbert schemes, and the McKay correspondence

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ABSTRACT

In 1988 Macdonald discovered a new family of symmetric polynomials, which have since found important uses in geometry, harmonic analysis, representation theory, and physics. In algebraic combinatorics, interest has centered on Macdonald's conjecture that certain coefficients $K_{\lambda\mu}(q,t)$ arising in his theory are polynomials with non-negative integer coefficients. My work on this conjecture led me to discover a fundamental connection between Macdonald polynomials and the geometry of Hilbert schemes of points in the plane, which explains various aspects of Macdonald's theory, as well as some interesting combinatorial conjectures on the space of "doubled" harmonics for the symmetric groups S_n.

The geometric setting for Macdonald polynomials is related to the remarkable conjectured ``McKay correspondence" between characters of a finite group $G \subset GL(V)$ and cohomology of certain nice desingularizations of V/G. The results yield some high-dimensional cases of a conjecture of Nakamura on G-Hilbert schemes, and suggest how the phenomena involving Maconald polynomials, Hilbert schemes, and doubled harmonics might extend to Coxeter groups other than S_n.