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*Is there a geometry of difference equations?*

A *difference equation* is an algebraic equation in several variables, involving an additional symbol $\sigma$ intended to denote a field automorphism. A difference variety is the set of solutions (in a field with a distinguished automorphism) to a system of difference equations.

A classical example is the equation $\sigma(x) = x$, coupled with algebraic equations $f(x) = 0$ defining a variety $V$. If $\sigma$ is taken to be the automorphism $x \mapsto x^p$ in a field of characteristic $p$, the corresponding difference variety coincides with the set of points of $V$ over the finite field $\mathbf{GF}(p)$. If $\sigma$ is the operator $f(x) \mapsto f(x+1)$ on a field of functions, the solutions are the periodic maps into $V$.

I will try to describe the general geography of the category of difference varieties. It contains (in two ways) the category of algebraic varieties. A number of model theoretic results suggest that the most interesting sites lie in a small but proper neighborhood of those described by algebraic geometry.