Fat 3-Spheres, 4-Polytopes and 5-Lattices

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The following three classes contain very similar objects -- in a topological resp. geometric resp. combinatorial model:

- 3-dimensional CW-spheres with the intersection property,
- 4-dimensional convex polytopes, and
- Eulerian lattices of rank 4.

$$\frac{f_1+f_2}{f_0+f_3}$$

We introduce and study the parameter of *fatness*, for these three classes -- which seems to be a key indicator to show how little we know. So, it is not clear whether fatness is bounded at all on any of these classes. Here we construct examples of

- rational 4-dimensional convex polytopes of fatness larger than 5-ε,
- 4-dimensional convex polytopes of fatness larger than 5.01, and
- 3-dimensional CW-spheres with the intersection property of fatness larger than 6-ε.

This implies counter-examples to conjectured f-vector inequalities of Bayer (1988) and of Billera & Ehrenborg (1999).

Most of our examples are constructed using the ``Eppstein construction'': as the convex hull of a 4-polytope with all ridges tangent to S^3, and its polar. This construction has a close connection with ball packings in S^3. Their study should lead to an infinite family of 2-simple 2-simplicial polytopes.

(Joint work with David Eppstein, UC Irvine)