Math 1B - Summer '09
Instructor: Daniel Halpern-Leistner
Final - 8/14/09 Name: ______________________

Complete the following problems on the additional sheets of paper provided (Clearly label your solutions by problem number). Show all work clearly, and **circle your final answers**. Cross out any work that you do not want graded. You may not use calculators, but you may use the formula sheet that I have provided. There are 5 problems for a total of 105 points. You will have 110 minutes to complete the exam.

1. ____________ / 20
2. ____________ / 30
3. ____________ / 15
4. ____________ / 20
5. ____________ / 20

Total ________________ / 105

Some basic MacLaurin series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^n}{n}$$

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

where $\binom{k}{n} = \frac{k(k-1)\cdots(k-n+1)}{n!}$ for $n > 0$ and $\binom{k}{0} = 1$.

PROBLEM ONE

(a) (10 points) Solve the differential equation

$$y'\sqrt{x^2-2x+2} = y^2$$

(b) (10 points) Compute the integral

$$\int_1^{\infty} \frac{x-1}{(1+x^2)(x+x^2)}$$

PROBLEM TWO

Decide if the following series diverge, converge absolutely, or converge conditionally

(a) (10 points) $\sum_{n=1}^{\infty} \frac{\sin(n)n}{\sqrt{n^5+7n}}$

(b) (10 points) $\sum_{n=1}^{\infty} \sqrt{\frac{1}{n} - \sin(\frac{1}{n})}$

(c) (10 points) Find the interval of convergence of the power series. For the endpoints, just determine if it converges or diverges and mention which test you would use to show it (including the series you would compare it to if you are using a comparison test).

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n3^n}(2x-1)^n$$

PROBLEM THREE (15 points)

Find the first four terms (i.e. compute $T_3(x)$) of the power series solution of the differential equation

$$y'' + y' + x^2 y = 3y, \qquad y(0) = 2, y'(0) = 1$$

PROBLEM FOUR

(a) (10 points) Use a power series to compute the integral $f(a) = \int_0^a e^{-\frac{x^2}{2}} dx$ as a function of a.

(b) (10 points) What is $T_3(a)$ for this function? Assuming $|a| < 1$, how small should a be so that $T_3(a)$ approximates $f(a)$ to within an error of .0001?

PROBLEM FIVE

Consider the differential equation

$$y'' + 2y' + (1 + k)y = 2e^{-x}$$

(a) (10 points) Solve the equation when $k = 0$.

(b) (10 points) For which values of k are the solutions of the complementary (homogenous) equation guaranteed to go to 0 as $x \to \infty$?

Thanks for a great summer! Use this page for extra space: