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Spring 2006, Math 104
First Midterm

10 Feb., 2006
3:10-4:00 PM

1. (32 points, 8 points each.) Complete the following definitions. You may use, without defining them, terms or symbols that Rudin defines before he defines the word or symbol asked for. Your definitions do not have to have exactly the same wording as those in Rudin, but for full credit they should be clear, and mean the same thing as his.

(a) An *ordered field* is a field F given with an ordering $<$ which (in addition to the conditions defining "field" and "ordered set") satisfies

(b) In the field of complex numbers, regarded as consisting of pairs (a, b) of real numbers, the real numbers are identified with the subfield consisting of

(c) Two sets A and B are said to have the same *cardinality* (or in Rudin, to be *equivalent,* written $A \sim B$) if

(d) If X is a metric space and E a subset of X, then a point $p \in E$ is said to be an *interior point* of E if

2. (32 points, 8 points each.) For each of the items listed below, either *give an example* with the properties stated, or give a brief reason why *no such example exists.*

If you give an example, you do *not* have to prove that it has the property stated; however, your examples should be specific; i.e., even if there are many objects of a given sort, you should name a particular one. If you give a reason why no example exists, don't worry about giving reasons for your reasons; a simple statement will suffice. (a) An ordered field which does not have the least upper bound property.

(b) Two members of the set of extended real numbers whose sum is not defined.

(c) A bijection (one-to-one correspondence) between the field Q of rational numbers and the field R of real numbers.

(d) A set of real numbers which is neither open nor closed in R.

3. (18 points) Prove that $\sup \{x+y-z \mid x, y, z \in R, \ x < y < z < 0\} = 0$.

4. (18 points) For any subset E of a metric space X, let us write $L(E)$ for the set of all limit points of E in X. Prove that if $E_1, E_2, \ldots, E_n, \ldots$ is a sequence of subsets of X, then $L(\bigcap_{n=1}^{\infty} E_n) \subseteq \bigcap_{n=1}^{\infty} L(E_n)$.