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100 Lewis Hall

Fall 2005, Math 55
First Midterm

19 Sept., 2005
1:10-2:00

1. (30 points, 10 points each.) Short answer questions. A correct answer will get full credit whether or not work is shown. An incorrect answer may get partial credit if work is given that follows a basically correct method.

(a) Give the truth table for the proposition $(q \to p) \wedge p$. (Your table must show columns for p, for q and for this compound proposition. It may or may not have other columns.)

(b) Let $f: \mathbf{R} \to \mathbf{R}$ be given by the rule $f(x) = 3x+1$, and $g: \mathbf{R} \to \mathbf{R}$ by the rule $g(x) = 4x+1$. Then $f \circ g: \mathbf{R} \to \mathbf{R}$ is given by the rule

(c) Write in mathematical symbols the statement that every real number which is not an integer lies between some integer and its successor (where the successor of an integer n means the integer $n+1$. If your statement is long, you don't have to put it all on one line.)

2. (24 points, 8 points each.) Complete the following definitions. Your definitions do not have to have exactly the same wording as those in the text, but for full credit they should be clear, and be equivalent to those.

(a) If X and Y are sets, and $f: X \to Y$ is a function, then the *graph* of f is defined to be the set . . .

(b) Let I be a set, and suppose that for each $i \in I$ we are given a set A_i. Then $\bigcup_{i \in I} A_i$ denotes . . . (For full credit, use set-builder notation rather than words.)

(c) If $f: \mathbf{R} \to \mathbf{R}$ and $g: \mathbf{R} \to \mathbf{R}$ are functions, then one says that $f(x)$ is $\Omega(g(x))$ (in words, "$f(x)$ is big-Omega of $g(x)$") if . . .

3. (30 points, 15 points each.) Short proofs. In giving the proofs asked for below, you may call upon definitions and results proved or asserted in the text. You do not have to use the formal names of methods of proof, or any standardized format, as long as your arguments are clear and logically sound.

(a) Suppose X, Y and Z are sets, where X and Y are subsets of a set S, and suppose $f: S \to Y$ a function. Prove that $f(X-Z) \supseteq f(X) - f(Z)$.

(b) Suppose that $f: \mathbf{R} \to \mathbf{R}$ and $g: \mathbf{R} \to \mathbf{R}$ are functions such that $f(x)$ is $O(g(x))$. Prove that $f(\log(|x|))$ is $O(g(\log(|x|)))$. (Recall that in this course, "log" means the logarithm to the base 2. You may use well-known facts about the logarithm function.)

4. (16 points) Write in pseudocode an algorithm "*intersect*" which takes two sequences of real numbers $a_1, \ldots, a_m$ and $b_1, \ldots, b_n$, where the elements of each sequence are distinct (i.e., for $i \neq j$, one has $a_i \neq a_j$ and $b_i \neq b_j$) and creates a sequence $c_1, \ldots, c_r$ of distinct real numbers such that

$$\{c_1, \ldots, c_r\} = \{a_1, \ldots, a_m\} \cap \{b_1, \ldots, b_n\}.$$

That is, after the algorithm has run, r should equal the number of elements in that intersection, and $c_1, \ldots, c_r$ should be the distinct elements in that intersection. For full credit you should use only the basic operations given in the text, and follow the format for pseudocode specified there. (However, you are not expected to give any equivalent of the changes between **bold** *italic* and roman font that the text uses.)