MATH 104 MIDTERM — OCTOBER 7, 2003

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All answers should fit on this page. Please do not use any auxiliary piece of paper. Good luck!

(1) Consider the sequences

$$A = (2, 2.2, 2.22, 2.222, 2.2222 \ldots);$$
$$B = (2, 2.1, 2.01, 2.001, 2.0001, \ldots);$$
$$C = (2, 2, 2, 2, 2, \ldots);$$
$$D = (3, 3.1, 3.14, 3.141, 3.1415, \ldots);$$
$$E = (4, 3.2, 3.15, 3.142, 3.1416, \ldots);$$
$$F = (1, 2, 3, 4, 5, \ldots);$$
$$G = (s_n = \sin(n));$$
$$H = (s_n = \frac{\sqrt{n} - 1}{\sqrt{n} + 1}).$$

Answer without proof:

- Which of these sequences are convergent?
- Which of these have the same limit?
- What are their subsequential limits (also known as *accumulation points*)?

(2) Construct (without proof) three examples of convergent sequences (s_n), (v_n) with $\lim s_n = 0$ and $\lim v_n = +\infty$ illustrating the possible cases

(a) $\lim(s_n \cdot v_n) = 0$;

(b) $\lim(s_n \cdot v_n) = 1$;

(c) $\lim(s_n \cdot v_n) = +\infty$.

(3) Let $A > 1$ be any real number. Consider the sequence (s_n) given by $s_1 = A$ and

$$s_{n+1} = \frac{A - 1}{A} s_n + \frac{1}{s_n}.$$

Give a (formal) proof that (s_n) is a convergent sequence. What is its limit?