Math 128a Midterm Exam 1

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September 26, 2003

NAME (printed) : _____

(Family Name) (First Name)

Signature : _____

Student Number : _____

(1) Do NOT open this test booklet until told to do so

(2) Do ALL your work in this test booklet

(3) Show ALL your work

(4) Check that there are 6 problems and 7 pages (including this one)

(5) NO CALCULATORS

(6) No pushing, biting, or hitting

1	2	3	4	5	6	TOTAL

1 a: (4 pts) Let $f(x)$ be a continuous function on $[2,3]$. Further, assume that for all $x \in [2,3]$ that $f(x) \in [2,3]$. Prove that $f(x)$ has a fixed point $p \in [2,3]$.

b: (3 pts) Further assume to part **(a)** that $|f'(x)| \leq \frac{1}{11}$ for all $x \in [2,3]$. How many steps of the fixed point method are needed to find the fixed point of $f(x)$ between $[2,3]$ to an accuracy of 10^{-3}.

2 a: (3 pts) Let $g(x) = ax + b$ for some fixed non-zero constants a and b. How many steps of Newton's method are needed to find a root of $g(x)$ to an accuarcy of 10^{-6}. Why?

b: (4 pts) Give an example of a, b and c, such that when using three digits rounding,

$$a + (b + c) \neq (a + b) + c$$

3 a: (3 pts) Define what it means for a sequence $\{p_n\}_{n=0}^{\infty}$ to converge linearly to p.

b: (3 pts) Let $p_n = 10^{-3^n}$. What order of convergence does p_n have?

c: (4 pts) Give two situations where you would use Bisection method over Newton's method.

4 a: (3 pts) Let $p_n \to p$. Give two conditions that are necessary to use accelerated convergence to compute $\hat{p}_n$.

b: (2 pts) Given $p_1 = 9, p_2 = 7, p_3 = 4$, compute $\hat{p}_1$.

5 a: (3 pts) Explain how to derive Secant Method from Newton's method.

b: (2 pts) Let $f(x) = x^4 - 17$. Compute the first two steps of Bisection method on the interval $[0, 4]$.

6 a: (4 pts) Use Horner's method to evaluate $p(x) = x^3 + x^2 - x - 1$ at $x = 2$.

b: (2 pts) Using the results of part (**a**), find $q(x)$ where

$$p(x) = (x - 2)q(x) + c$$

where c is your answer in part (**a**).