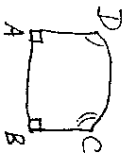


Math 130 Final H. Wu
Dec 17, 2002

Hilbert Plane

- In Hilbert plane + (P), (a) prove that any two altitudes of a triangle must meet, and (b) prove that all three altitudes of a triangle are concurrent, if it meets at a point. (30 points)
- In a Hilbert plane, prove AAS. More precisely, suppose two triangles, $\triangle ABC$ and $\triangle A'B'C'$ satisfy $\overline{AB} \cong \overline{A'B'}$, $\angle A \cong \angle A'$ and $\angle C \cong \angle C'$, then $\triangle ABC \cong \triangle A'B'C'$. (30 points)
- In Hilbert plane + (A), if there exists one triangle with angle sum $\cong 180$, then (P) must hold. (20 points)
- In a Hilbert plane, suppose the top angles $\angle C$ and $\angle D$ of a Saccheri quadrilateral are of distance, then $\overline{AB} > \overline{DC}$. (20 points)



Coordinate Geometry

- Prove that through a point P not on a line ℓ , there is one and only one line L such that $L \perp \ell$. (30 points)
- Prove SAS for similar triangles. More precisely, suppose

$\triangle ABC$ and $\triangle A'B'C'$ are given such that $\angle A \cong \angle A'$ and \bullet .
 Then $\triangle ABC \sim \triangle A'B'C'$. (20 points)

$$\frac{|\overline{AB}|}{|\overline{A'B'}|} = \frac{|\overline{AC}|}{|\overline{A'C'}|}$$

Classical Euclidean Geometry

- In $\triangle ABC$, let $D \in AB$, $E \in AC$. Prove $\overline{DE} \parallel \overline{BC}$ is an anti-parallel to \overline{BC} iff $\overline{DE} \parallel$ the tangent to the circumcircle of $\triangle ABC$ at A (c' b' in the picture). (30 points)
- Let the tangents to the circumcircle of $\triangle ABC$ at the vertices meet at A', B', C' . Prove that AA' bisects every anti-parallel to \overline{BC} . (20 points)

