MATH 185 – MIDTERM #1 Spr 02

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INSTRUCTIONS: Answer each question on a separate sheet of paper, and write your name on each page. Each problem counts equally.

Problem #1. Write the following in $x + iy$ form:

$$\left(\frac{\sqrt{2}}{i-1}\right)^{10}.$$

Problem #2. Compute

$$\int_C \frac{1}{z^2 + 2z - 3}\, dz,$$

where C is the positively oriented circle of radius 2, centered at the origin.

Problem #3. Let

$$z = \sin w.$$

Solve for w in terms of z, to derive the formula

$$w = \sin^{-1} z = -i \log(iz + (1 - z^2)^{\frac{1}{2}}).$$

Problem #4. Assume $f : \mathbb{C} \to \mathbb{C}$ is differentiable at a point z_0. State and prove the *Cauchy–Riemann equations.*

Problem #5. Let u and v be harmonic functions, and suppose that v is the conjugate of u. Show that

$$\frac{u}{u^2 + v^2} \text{ and } \frac{-v}{u^2 + v^2} \text{ are harmonic,}$$

assuming $u^2 + v^2 \neq 0$.

Problem #6. Let f be an entire function, and suppose that for all points x on the real axis, $f(x)$ is purely imaginary.

Show that

$$\overline{f(z)} = -f(\bar{z}) \quad \text{for all } z \in \mathbb{C}.$$

HINT: Let $g(z) = if(z)$; so that $g(x)$ is real for points x on the real axis.