NAME _____

STUDENT ID NUMBER _____

TA's name or section number _____

MATH 1B Final Exam Fall 2001

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There are 450 points altogether.

The first 15 questions are multiple choice, each worth 15 points. Choose the most correct answer to each question and mark the corresponding box in the grid ON THE BACK OF THIS PAGE. Mark only one box per question. No partial credit.

TA use only:

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1

Question	a	b	c	d	e
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Multiple choice questions:

1) Which of the following is correct for any convergent series $\sum_{n=1}^{\infty} a_n$ with positive terms?

(a) $\sum_{n=1}^{\infty} (a_n + 1)$ converges.

(b) $\sum_{n=1}^{\infty} a_n^2$ converges.

(c) $\sum_{n=1}^{\infty} 1/a_n$ converges.

(d) $\sum_{n=1}^{\infty} \sqrt{a_n}$ converges.

(e) $\sum_{n=1}^{\infty} \sqrt{a_n}$ diverges.

2) The recurring decimal 0.12121212.... is the rational number

(a)3/11

(b)3/22

(c)1/12

(d)3/25

(e)4/33

3) Which of the following strategies is most likely to succeed to find a particular solution of the differential equation $y'' - y = \dfrac{1}{x}$, for $x \neq 0$?

(a) Try a solution of the form $\dfrac{A}{x} + B$ for some constants A and B.

(b) Try a power series solution $\sum_{n=0}^{\infty} a_n x^n$.

(c) Try a solution of the form $v_1(x)e^x + v_2(x)e^{-x}$ for some functions v_1 and v_2.

(d) Try a solution of the form $\frac{A}{x}e^x + \frac{B}{x^2}e^x$ for some constants A and B.

(e) Try a solution of the form $\frac{A}{x}e^x + \frac{B}{x}e^{-x}$ for some constants A and B.

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(4) Which of the following is most correct for the complex numbers Z and W marked with x's in the picture of the complex numbers below? (The dashed circle represents the unit circle - that is to say all complex numbers of modulus 1.)

(a)$Z = W + i$

(b)$Z = W^2$

(c)$W = Z^2$

(d)$Z = W - 1$

(e)$Z = 2W$
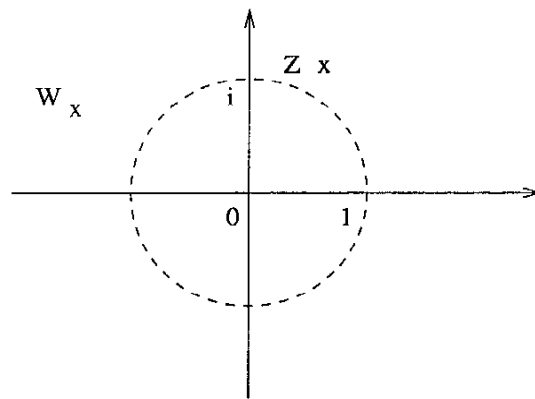
5)Which of the following is true for any sequence $\{a_n\}$ with $\lim\limits_{n\to\infty} a_n = 4$?

(a) There is an $N > 0$ for which $a_n < 2$ for all $n \leq N$.

(b) There is an N for which $|a_n - 4| < 1$ for all $n \geq N$.

(c) $\lim\limits_{n\to\infty}(a_n + a_{n+1}) = \infty$.

(d) For no value of n is a_n bigger than 300.

(e)For any $\epsilon > 0$ there is an N with $|a_n - 4| > \epsilon$ for all $n \geq N$.

6)The integral $\displaystyle\int_2^\infty \frac{1}{x^2 - 6x + 9}$ is

(a)divergent

(b)1/2

(c)1

(d)2

(e)$ln(3)$

4

7)The general solution to the differential equation $y'' + 2y' + 5y = 0$ is

(a) $c_1 e^{3x} + c_2 e^{-x}$

(b) $c_1 e^{-3x} + C_2 e^x$

(c) $e^{\sqrt{5}x}(c_1 \cos \frac{x}{\sqrt{2}} + c_2 \sin \frac{x}{\sqrt{2}})$

(d) $Ae^{-x} \cos(2x + \phi)$

(e) $c_1 x e^{3x} + c_2 e^{3x}$

8) Which of the following integrals gives the area of the surface obtained by rotating the curve $y = e^x$ for y between $1/e$ and e about the line $x = 2$?

(a)$2\pi \int_{-1}^{1} (2 - e^x)\sqrt{1 + e^{2x}} \, dx$

(b)$2\pi \int_{1/e}^{e} (e^x - 2)\sqrt{1 + e^{2x}} \, dx$

(c)$2\pi \int_{1/e}^{1} (2 - ln(x))\sqrt{1 + e^{2y}} \, dy$

(d)$2\pi \int_{1/e}^{e} (2 - ln(y))\sqrt{1 + \frac{1}{y^2}} \, dy$

(e)$2\pi \int_{-1}^{1} (x - 2)\sqrt{1 + e^{2x}} \, dx$

9) To integrate the function $\dfrac{x^3}{x^3 - 1}$ by partial fractions one should try to express it in the form

(a) $1 + \dfrac{A}{x - 1} + \dfrac{Bx + C}{x^2 + x + 1}$

(b)$\dfrac{A}{x^3} - \dfrac{B}{x^2} + \dfrac{C}{x}$

(c)$\dfrac{A}{x^3} + \dfrac{B}{x^2} - \dfrac{C}{x}$

(d)$\dfrac{A}{x - 1} + \dfrac{Bx + C}{x^2 + x + 1}$

(e)$\dfrac{A}{x - 1} - \dfrac{Bx + C}{x^2 + x + 1}$

5

10) A solution $y = g(x)$ to the equation $dy/dx = f(x,y)$ is given by the curve sketched in the figure below. Which of the following is most likely to be correct concerning the approximation y_1 to $g(1)$ that would be obtained by Euler's method starting at $x = 0$ with step size 0.1?

(a) $y_1 = g(1)$
(b) $y_1 > g(1)$
(c) $y_1 < g(1)$
(d) $y_1 > g(1) + 0.1$
(d) $y_1 > g(1) + 1$
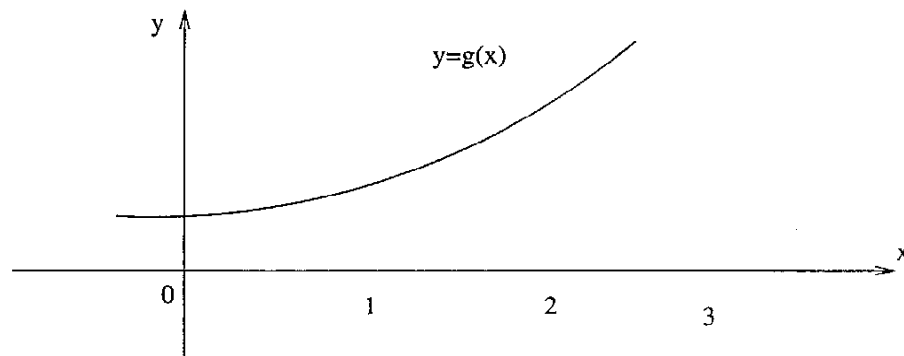
11) The complex number e^{2+i} is equal to

(a) $e^{\sqrt{5}}(\cos(\tan^{-1}(1/2)) + i\sin(\tan^{-1}(1/2)))$
(b) $e^{\sqrt{5}}(\cos(\tan^{-1}(2)) + i\sin(\tan^{-1}(2)))$
(c) $-e^2$
(d) $e^2(\cos 1 + i\sin 1)$
(c)   a shoe

12) Consider the differential equation $\frac{dP}{dt} = 5P(1-P)$. Which of the following is correct?
(a) If $P(0) = 0.5$, then $P(t) < 1$ for all positive t.
(b) If $P(0) = 0.5$ then $P(1) = 1$.
(c) If $P(2) = 6$ then $P(t)$ is increasing when $t = 7$.
(d) If $P(0) = 0.5$ then $P(t) = 0.5$ for all t.
(e) Squash is the most popular sport in the United States.

6

13) Which of the following is correct concerning a linear homogeneous second order differential equation with constant coefficents?

(a) The boundary value problem always has a unique solution.

(b) The initial value problem always has a unique solution.

(c) The initial value problem may not have a solution but if one exists it is unique.

(d) The boundary value problem may not have a solution but if one exists it is unique.

(e) The boundary value problem always has a solution but there may be infinitely many different ones.

14) Suppose the sequence (a_n) is such that $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ converges. It then follows that:

(a) $lim_{n\to\infty} a_n = 0$

(b) $\sum_{n=1}^{\infty} a_n$ converges absolutely.

(c) $\sum_{n=1}^{\infty} a_n$ converges.

(d) $\sum_{n=1}^{\infty} \frac{a_n}{4^n}$ converges absolutely.

(e) $\sum_{n=1}^{\infty} a_n$ diverges.

15) $lim_{x\to 0} \quad \dfrac{e^x - \cos^2 x}{\sin^2 x - ln(1-x)}$ equals

(a) ∞
(b) 0
(c) 1
(d) -1
(e) 2

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The next six questions are *not* multiple choice. Show your reasoning and give your answers in the space provided.

1.(50 points)

Find two linearly independent solutions to the differential equation

$$y'' + x^2 y = 0$$

2.(40 points)

A vat has a volume of 100 liters. It initially contains 50 liters of pure water. Brine with a concentration of 1 gram per liter begins to flow into the vat at a rate of 2 liters per minute. The mixed solution escapes through a leak at a rate of 1 liter per minute. How much salt is there in the vat when it begins to overflow?

3.(30 points) Solve the initial value problem

$$y'' + y' + y = 0, \quad y(0) = 0, y'(0) = 1$$

4. (a)(20 points) Is the series $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2 - n}$ convergent? If so find its sum.

(b)(10 points) Suppose f is a continuous positive strictly decreasing function for $x \geq 1$ and $a_n = f(n)$.

By drawing a picture, rank the following in increasing order:

$$\int_1^6 f(x)\ dx \qquad \sum_{i=1}^5 a_i \qquad \sum_{i=2}^6 a_i.$$

11

12

5)(30 points) Find a particular solution of the differential equation

$$y'' - y' - y = e^{2x} + 1$$

6)(45 points-15 each) Evaluate the following integrals:

(i) $\int_{0}^{1} x\sqrt{1-x^4}\,dx$

(ii) $\int_{1}^{\infty} \frac{1}{x^{\sqrt{2}}}\,dx$

$(iii) \int x^3 e^{x^2} dx$